Available online at www.sciencedirect.com
ScienceDirect
Applied Mathematics and Computation 198 (2008) 35–43
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

A numerical method for solving a class of functional and two dimensional integral equations

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Abstract

In this paper, the Chebyshev collocation method has been used to solve the functional integral equation of the first and second kind. Also the numerical solution of the two dimensional Fredholm–Volterra integral equations (F–VIE) of the second kind is considered. The Chebyshev collocation method transforms any integral equation into a system of linear algebraic equations. In this method the Chebyshev expansion coefficients of the solution is obtained. Finally some examples show the accuracy of this method.

Keywords: Chebyshev collocation method; Fredholm–Volterra integral equations; Functional integral equation

1. Introduction

As we know the Chebyshev polynomials are one of the best orthogonal polynomials that have important role particularly in numerical analysis. Also, the numerical methods for solving fuzzy/non-fuzzy integral equations are treated in the literature [1–4,6,7]. In this paper, we use the following new definition [9,10] for the integral equations such that Volterra and Fredholm integral equations.

Fredholm functional integral equation of the second kind and first kind are given by

$$y(s) + p(s)y(h(s)) + \lambda \int_a^b k(s,t)y(t)\mathrm{d}t = g(s), \quad a \leqslant s \leqslant b,$$

and

$$p(s)y(h(s)) + \int_a^b k(s,t)y(t)\mathrm{d}t = g(s), \quad a \leqslant s \leqslant b,$$

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0096-3003/$ - see front matter
doi:10.1016/j.amc.2007.08.051

respectively. Also Volterra functional IE of the second and first kind are given by

$$y(s) + p(s)y(h(s)) + \lambda \int_a^s k(s,t)y(t)\mathrm{d}t = g(s), \quad a \leqslant s \leqslant b,$$

and

$$p(s)y(h(s)) + \int_a^s k(s,t)y(t)\mathrm{d}t = g(s), \quad a \leqslant s \leqslant b.$$

Obviously, F–VIE of the first and second kind are special case of these equations (by putting $p(s) = 0$). Here we assume that all functions are defined in interval $[-1,1]$, otherwise by using suitable change of variable we obtain this interval, which is the domain of the Chebyshev polynomials of the first kind. In this paper, we approximate the solution of functional IE using Chebyshev basis and the method of collocation with Chebyshev points. This method had been used for systems of linear IE [5]. First we consider Fredholm functional integral equations of the second and first kind as

$$y(s) + p(s)y(h(s)) + \int_{-1}^1 k(s,t)y(t)\mathrm{d}t = g(s), \quad -1 \leqslant s \leqslant 1, \tag{1}$$

and

$$p(s)y(h(s)) + \int_{-1}^1 k(s,t)y(t)\mathrm{d}t = g(s), \quad -1 \leqslant s \leqslant 1. \tag{2}$$

The aim of our method is to get solution as truncated Chebyshev series defined by

$$y(s) \simeq y_N(s) = \sum_{j=0}^N a_j T_j(s), \tag{3}$$

where $T_j(s)$ denote the Chebyshev polynomials of the first kind, a_j are unknown coefficients and N is any positive integer. We assume the kernel and the solution of this equations can be expressed as a truncated Chebyshev series. Then we can write (3) in the matrix form

$$y_N(s) = \mathbf{T}(s)\mathbf{A}, \tag{4}$$

where

$$\mathbf{T}(s) = [T_0(s)\ T_1(s)\ \cdots\ T_N(s)],$$
$$\mathbf{A} = [a_0\ a_1\ \cdots\ a_N]^{\mathrm{T}}.$$

The method of collocation solves the FIE (1) using the approximation (3) through the equations

$$r_N(s_i) = y_N(s_i) + p(s_i)y_N(h(s_i)) + \int_{-1}^1 k(s_i,t)y_N(t)\mathrm{d}t - g(s_i) = 0, \tag{5}$$

for collocation points $s_i = \cos(i\pi/N) \in [-1,1]$, $i = 1,\ldots,N$. Similarly kernel function $k(s,t)$ can be expressed as a truncated Chebyshev series for each s_i in the form

$$k(s_i,t) \simeq k_N(s_i,t) = \sum_{r=0}^{N}{}'' k_r(s_i)T_r(t), \tag{6}$$

where double prime denotes that the first and the last terms are halved, the $k_r(s_i)$ are determined by means of Cleanshaw–Kurtis rule, [7] as follows:

$$k_r(s_i) = \frac{2}{\pi}\int_{-1}^1 \frac{k(s_i,t)T_r(t)}{(1-t^2)^{0.5}}\mathrm{d}t \simeq \frac{2}{\pi} \times \frac{\pi}{N}\sum_{m=0}^{N}{}'' k(s_i,t_m)T_r(t_m) = \frac{2}{N}\sum_{m=0}^{N}{}'' k(s_i,t_m)T_r(t_m),$$

where $t_m = \cos(m\pi/N)$ for $m = 0,1,\ldots,N$ and $k_N(s_i,t)$ can be represented in matrix form

$$k_N(s_i,t) = \mathbf{K}(s_i)\mathbf{T}'(t), \tag{7}$$