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Robust reliable H_∞ control for uncertain nonlinear systems via LMI approach

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Abstract

This paper investigates the robust reliable H_∞ control for a class of uncertain nonlinear system. At first, a criterion for nonlinear system without uncertainties is proposed to guarantee global exponential stabilization and disturbance attenuation. Next, the criterion for uncertain nonlinear systems with parameter uncertainties is obtained by simple derivation. Linear matrix inequality (LMI) optimization approach is used to design the robust reliable H_∞ state feedback control. Two numerical examples are given to illustrate the effectiveness of the main results.

Keywords: Reliable control; H_∞ control; Exponential stabilization; LMI approach; Uncertain systems

1. Introduction

In the recent years, sensor and actuator failures of linear systems are included to consider in theory analysis and feedback control. Reliable control is introduced to tolerate the failures of sensor and actuator and maintains the system stability and performance. Many approaches had been proposed to design reliable control in this situation of sensor and actuator outages [1–4]. Algebraic Riccati-equation approach is presented to guarantee closed-loop stability and H_∞ performance in some admissible component failures [3]. In [1], Hamilton–Jacobi equation approach is used to design reliable control for nonlinear systems. On the other hand, system models always contain some uncertain elements and nonlinearities; these uncertainties and nonlinearities may be due to additive unknown noise, environmental influence, poor plant knowledge, and limitation of actuators or sensors. Hence robust reliable control technology is developed to stabilize the uncertain linear or nonlinear systems with sensor and actuator failures [1].

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Nomenclature

A^T transpose of matrix A
$L_2[0,\infty)$ space of square integrable functions on $[0,\infty)$
$\|x\|$ Euclidean norm of vector x
$\|f\|_2$ $\sqrt{\int_0^\infty \|f(t)\|^2 \mathrm{d}t}, f(t) \in L_2[0,\infty)$
diag$[a_i]$ diagonal matrix with the diagonal elements a_i, $i = 1, \ldots, n$
$P > 0$ (respectively, $P < 0$) P is a positive (respectively, negative) definite symmetric matrix
$A \leqslant B$ B–A is a positive semi-definite symmetric matrix
$\lambda_{\min}(P)$ minimal eigenvalue of real symmetric matrix P
$\lambda_{\max}(P)$ maximal eigenvalue of real symmetric matrix P
I unit matrix
$\begin{bmatrix} A & B \\ * & C \end{bmatrix}$ $*$ represents the symmetric form of matrix; i.e. $* = B^T$

Over the past few decades, the H_∞ control problem for uncertain systems with disturbance inputs has been an active topic in control system theory and application [1,4–7]. The H_∞ control is proposed to reduce the effect of the disturbance input on the regulated output to within a prescribed level. There are many approaches in dealing the H_∞ control problem. Riccati-equation approach was proposed to design H_∞ control for time-delay system [5]. Riccati-equation and Hamilton-Jacobi equation approaches are difficult to find their feasible solutions and minimization of H_∞-norm bound (γ). In [4], the LMI approach had been used to design reliable H_∞ control for a given H_∞-norm bound (γ). In [6,7], LMI optimization approach is an efficient method and had been used to find the minimization of H_∞-norm bound in their considered systems. In this paper, LMI optimization approach will be used to design the reliable H_∞ control for uncertain nonlinear system. Global exponential stabilization will be guaranteed by the reliable feedback control. Two numerical examples will be illustrated to show the main result of this paper.

2. Reliable H_∞ control for unperturbed nonlinear systems

Consider the following nonlinear system:

$$\dot{x}(t) = Ax(t) + B_u u^f(t) + B_w w(t) + f(x(t)), \quad t \geqslant 0, \tag{1a}$$

$$x(0) = x_0, \tag{1b}$$

$$z(t) = Cx(t) + D_u u^f(t) + D_w w(t), \tag{1c}$$

where $x \in \Re^n$ is the state at time t, $u^f \in \Re^n$ is the control input of actuator fault, $w \in \Re^l$ is the disturbance input, $z \in \Re^q$ is the regulated output, $x_0 \in \Re^n$ is the initial state. $A \in \Re^{n\times n}$, $B_u \in \Re^{n\times m}$, $B_w \in \Re^{n\times l}$, $C \in \Re^{q\times n}$, $D_u \in \Re^{q\times m}$, and $D_w \in \Re^{q\times l}$ are constant matrices, $f(x(t))$ is a nonlinear function satisfying

$$\|f(x(t))\| \leqslant \|Fx(t)\|, \tag{2}$$

where $F \in \Re^{n\times n}$ is a given constant matrix. The control input of actuator (or sensor) fault is described as follows:

$$u^f(t) = Ru(t), \tag{3a}$$

where R is the actuator fault matrix with

$$R = \mathrm{diag}[r_1, r_2, \ldots, r_m], \quad 0 \leqslant \underline{r}_i \leqslant r_i \leqslant \bar{r}_i, \quad \bar{r}_i \geqslant 1, \quad i = 1, 2, \ldots, m, \tag{3b}$$

$\underline{r}_i$ and $\bar{r}_i$, $i = 1, 2, \ldots, m$, are some given constants. $r_i = 0$ means that ith actuator completely fails, $r_i = 1$ means that ith actuator is normal.