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Some quadrature based three-step iterative methods for non-linear equations

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Abstract

In this paper, we present three-step quadrature based iterative methods for solving non-linear equations. The convergence analysis of the methods is discussed. It is established that the new methods have convergence order six, seven and eight respectively. Numerical tests show that the new methods are comparable with the well known existing methods and in many cases give better results. Our results can be considered as an improvement and refinement of the previously known results in the literature.

Keywords: Iterative methods; Three-step methods; Quadrature rule; Predictor–corrector methods; Non-linear equations

1. Introduction

Let us consider a single variable non-linear equation

$$f(x) = 0. \tag{1.1}$$

Finding zeros of a single variable non-linear equation (1.1) efficiently, is an interesting and very old problem in numerical analysis and has many applications in applied sciences.

In recent years, researchers have developed many iterative methods for solving Eq. (1.1). These methods can be classified as one-step, two-step and three-step methods, see [1–12]. These methods have been proposed using Taylor series, decomposition techniques, error analysis and quadrature rules, etc. Abbasbandy [1], Chun [3] and Grau and Diaz-Barrero [7] have proposed many two-step and three-step methods.

In this paper, we present three-step quadrature based iterative methods for solving non-linear equations. We prove that the new methods have order of convergence six, seven and eight, respectively. The methods and their algorithms are described in Section 2. The convergence analysis of these methods is discussed in Section 3. Finally, in Section 4, the methods are tested on numerical examples given in the literature. It was noted that the new methods are comparable with the well known existing methods and in many cases give better

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results. Our results can be considered as an improvement and refinement of the previously known results in the literature.

2. The iterative method

Weerakoon and Fernando [12], Nedzhibov [11] and Frontini and Sormani [5,6] have proposed various methods by the approximation of the indefinite integral

$$f(x) = f(x_n) + \int_{x_n}^{x} f'(t)\mathrm{d}t \tag{2.1}$$

using Newton Cotes formulae of order zero and one. We approximate, here however the integral (2.1) by rectangular rule at a generic point $\lambda x + (1-\lambda)z_n$ with the end-points x and z_n. We thus have

$$\int_{z_n}^{x} f'(t)\mathrm{d}t = (x - z_n)f'(\lambda x + (1-\lambda)z_n),$$

this gives

$$-f(z_n) = (x - z_n)f'(z_n) + \lambda(x - z_n)^2 f''(z_n). \tag{2.2}$$

From (2.2), we have

$$x = z_n - \frac{f(z_n)f'(z_n)}{f'^2(z_n) - \lambda f(z_n)f''(z_n)}. \tag{2.3}$$

From (2.3), for $\lambda = 0$, we have Newton's method and for $\lambda = 1/2$, we have Halley method.

This formulation allows to suggest many one-step, two-step and three-step methods. We suggest here, however the following three-step methods:

Algorithm 2.1. For a given initial guess x_0, find the approximate solution by the iterative scheme:

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}, \tag{2.4}$$

$$z_n = y_n - \frac{(x_n - y_n)f(y_n)}{f(x_n) - 2f(y_n)}, \tag{2.5}$$

$$x_{n+1} = y_n - \frac{f(y_n)f'(y_n)}{f'^2(y_n) - \lambda f(y_n)f''(z_n)}. \tag{2.6}$$

Algorithm 2.2. For a given initial guess x_0, find the approximate solution by the iterative scheme:

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}, \tag{2.7}$$

$$z_n = y_n - \frac{(x_n - y_n)f(y_n)}{f(x_n) - 2f(y_n)}, \tag{2.8}$$

$$x_{n+1} = z_n - \frac{f(z_n)f'(z_n)}{f'^2(z_n) - \lambda f(z_n)f''(z_n)}. \tag{2.9}$$

Algorithm 2.2 can be further modified by using an approximation for $f''(z_n)$ with the help of Taylor's expansion. By Taylor expansion of $f''(z_n)$ about x_n, we have

$$f''(z_n) \simeq f''(x_n)$$

(where the higher order derivatives are neglected).

By Taylor expansion, we have

$$f(z_n) \simeq f(x_n) + f'(x_n)(z_n - x_n) + \frac{1}{2}f''(x_n)(z_n - x_n)^2,$$