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A class of integral operators preserving subordination and superordination for meromorphic functions

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Abstract

The purpose of the present paper is to investigate several subordination- and superordination-preserving properties of a certain class of integral operators, which are defined on the space of meromorphic functions in the punctured open unit disk. The sandwich-type theorem for these integral operators is also presented. Moreover, we consider an application of the subordination and superordination theorem to the Gauss hypergeometric function.

Keywords: Meromorphic functions; Differential subordination; Differential superordination; Meromorphic starlike functions; Meromorphic convex functions; Integral operators; Best subordinant; Best dominant; Gauss hypergeometric function

1. Introduction

Let $\mathscr{H} = \mathscr{H}(\mathbb{U})$ denote the class of analytic functions in the open unit disk

$$\mathbb{U} = \{z : z \in \mathbb{C} \text{ and } |z| < 1\}.$$

For $a \in \mathbb{C}$, let

$$\mathscr{H}[a,n] := \{f : f \in \mathscr{H} \text{ and } f(z) = a + a_n z^n + a_{n+1} z^{n+1} + \cdots\}.$$

Let f and F be members of the analytic function class $\mathscr{H}$. The function f is said to be subordinate to F, or F is said to be superordinate to f, if there exists a function w analytic in $\mathbb{U}$, with

$$w(0) = 0 \quad \text{and} \quad |w(z)| < 1 \quad (z \in \mathbb{U}),$$

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such that

$$f(z) = F(w(z)) \quad (z \in \mathbb{U}).$$

In this case, we write

$$f \prec F \quad (z \in \mathbb{U}) \quad \text{or} \quad f(z) \prec F(z) \quad (z \in \mathbb{U}).$$

If the function F is univalent in $\mathbb{U}$, then we have (cf. [11])

$$f \prec F \quad (z \in \mathbb{U}) \Longleftrightarrow f(0) = F(0) \quad \text{and} \quad f(\mathbb{U}) \subset F(\mathbb{U}).$$

Definition 1 (Miller and Mocanu [11]). Let

$$\phi : \mathbb{C}^2 \to \mathbb{C}$$

and let h be univalent in $\mathbb{U}$. If p is analytic in $\mathbb{U}$ and satisfies the following differential subordination:

$$\phi(p(z), zp'(z)) \prec h(z) \quad (z \in \mathbb{U}), \tag{1.1}$$

then p is called a solution of the differential subordination. A univalent function q is called a dominant of the solutions of the differential subordination or, more simply, a dominant if $p \prec q$ for all p satisfying the differential subordination (1.1). A dominant $\tilde{q}$ that satisfies $\tilde{q} \prec q$ for all subordinants q of (1.1) is said to be the best dominant.

Definition 2 (Miller and Mocanu [12]). Let

$$\varphi : \mathbb{C}^2 \to \mathbb{C}$$

and let h be analytic in $\mathbb{U}$. If p and $\varphi(p(z), zp'(z))$ are univalent in $\mathbb{U}$ and satisfy the following differential superordination:

$$h(z) \prec \varphi(p(z), zp'(z)) \quad (z \in \mathbb{U}), \tag{1.2}$$

then p is called a solution of the differential superordination. An analytic function q is called a subordinant of the solutions of the differential superordination or, more simply, a subordinant if $q \prec p$ for all p satisfying the differential superordination (1.2). A univalent subordinant $\tilde{q}$ that satisfies $q \prec \tilde{q}$ for all subordinants q of (1.2) is said to be the best subordinant.

Definition 3 (Miller and Mocanu [12]). We denote by $\mathcal{Q}$ the class of functions f that are analytic and injective on $\overline{\mathbb{U}} \setminus E(f)$, where

$$E(f) := \left\{ \zeta : \zeta \in \partial\mathbb{U} \text{ and } \lim_{z \to \zeta} f(z) = \infty \right\},$$

and are such that

$$f'(\zeta) \neq 0 \quad (\zeta \in \partial\mathbb{U} \setminus E(f)).$$

Let Σ denote the class of functions of the form:

$$f(z) = \frac{1}{z} + \sum_{n=0}^{\infty} a_n z^n,$$

which are analytic in the *punctured* open unit disk

$$\mathbb{D} := \mathbb{U} \setminus \{0\}.$$

Let Σ^* and Σ_k be the subclasses of Σ consisting of all functions which are, respectively, meromorphic starlike and meromorphic convex in $\mathbb{D}$ (see, for details, [5,11]).