Profitless delays for a nonautonomous Lotka–Volterra predator–prey almost periodic system with dispersion ☆

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Abstract

This paper studies a nonautonomous Lotka–Volterra almost periodic predator–prey dispersal system with discrete and continuous time delays which consists of two-patches, the prey species can disperse among two-patches, but the predator species is confined to one patch and cannot disperse. By using comparison theorem and delay differential equation basic theory, we prove the system is uniformly persistent under some appropriate conditions. Further, by constructing suitable Lyapunov functional, we show that the system is globally asymptotically stable under some appropriate conditions. By using a new method and almost periodic functional hull theory, we show that the almost periodic system has a unique globally asymptotical stable strictly positive almost periodic solution. The conditions for the permanence, global stability of system and the existence, uniqueness of positive almost periodic solution depend on delays, so, time delays are "prof-itless". Finally, ecological conclusions and a particular case are given. These results are basically an extension of the known results for nonautonomous Lotka–Volterra systems.
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1. Introduction

The classical Lotka–Volterra type systems form a significant component of the models of population dynamics. Recently, delays [1–13] and diffusions [8,14–20] are extensively introduced into Lotka–Volterra type systems, which enriches biologic background. The effect of environment change in the growth and diffusion of a species in a heterogeneous habitat is a subject of considerable interest in the ecological literature [8,14–20].

Because of the ecological effects of human activities and industry, more and more habitats are broken into patches and some of them are polluted. In some of these patches, without the contribution from other patches, the species will go to extinction. Generally speaking, the delay differential equations exhibit much more complicated dynamics than ordinary differential equations since a time delay could cause a stable equilibrium to become unstable and cause the population to fluctuate. Negative feedback crowding or the effect of the past life history of the species on its present birth rate are common examples illustrating the biological meaning of time delays and justifying their use in these systems.

Since biological and environmental parameters are naturally subject to fluctuation in time, the effects of an periodically varying environment are considered as important selective forces on systems in a fluctuating environment. Therefore, on the one hand, models should take into account both the seasonality of the periodically changing environment and the effects of time delays (see [1–13]). However, on the other hand, in fact, it is more realistic to consider almost periodic system than periodic system. The most basic and important questions to ask for these systems in the theory of mathematical ecology are the persistence, extinctions, global asymptotic behaviors, and existences of coexistence states (for example, the positive equilibrium, strictly positive solution, positive periodic solution, periodic solution and almost periodic solution, etc.) of population (see [1–27]). In this paper, we will investigate the persistence, global asymptotic behaviors, strictly positive solution, strictly positive almost periodic solution of population in model. There is a vast amount of literature on periodic system (see [2,8–10,15–17,25–27]), however, papers on almost periodic dispersal systems with delays are few, and most of the papers on almost periodic system are based on the constructed appropriate Lyapunov functionals $V(t,x)$ which usually has the stronger conditions of theorem, because of the use of the theorem in [21] to satisfy $D^{+}V(t,x) \leqslant -cV(t,x(t))$ (where c is a positive constant) (see [11,12,21]). In this paper, by using a new method and almost periodic functional hull theorem, we directly analyze the right functional of almost periodic system, and obtain the new weak sufficient conditions with delays for the existence and uniqueness of almost periodic solution.

In this paper, we shall be firstly considered by the case of combined effects: dispersion and time delays. Namely, we investigate the following general nonautomous Lotka–Volterra type predator–prey dispersal system with discrete and continuous time delays which models the diffusion of the prey species x_i into two-patches connected by discrete dispersal.

$$\dot{x}_1(t) = x_1(t)\left[r_1(t) - a_{11}(t)x_1(t) - \int_{-\varrho_1}^{0} k_1(t,s)x_1(t+s)\,\mathrm{d}s - a_{13}(t)y(t)\right] + D_{21}(t)(x_2(t) - x_1(t)),$$

$$\dot{x}_2(t) = x_2(t)\left[r_2(t) - a_{22}(t)x_2(t) - \int_{-\varrho_2}^{0} k_2(t,s)x_2(t+s)\,\mathrm{d}s\right] + D_{12}(t)(x_1(t) - x_2(t)), \tag{1}$$

$$\dot{y}(t) = y(t)\left[-r_3(t) + a_{31}(t)x_1(t-\tau_1(t)) - a_{33}(t)y(t-\tau_2(t)) - \int_{-\varrho_3}^{0} k_3(t,s)y(t+s)\,\mathrm{d}s\right],$$

where x_1 and y are population density of prey species x and predator species y in patch 1, and x_2 is density of prey species x in patch 2. Predator species y is confined to patch 1, while the prey species x can disperse between two patches. $D_{ij}(t)$ $(i,j = 1,2, i \neq j)$ is dispersion rate of the prey species, τ_1 is the delay due to gestation, that is, mature adult predators can only contribute to the production of predator biomass. In addition, we have included the term $-a_{33}(t)y(t-\tau_2(t))$ in the dynamics of predator y to incorporate the negative feedback of predator crowding, the terms $\int_{-\varrho_i}^{0} k_i(t,s)x_i(t+s)\,\mathrm{d}s(i=1,2)$ and $\int_{-\varrho_3}^{0} k_3(t,s)y(t+s)\,\mathrm{d}s$ the effect of a period of the past life history of the species on its present birth rate. Suppose that $h(t)$ is an bounded function defined on R. Define $h^u = \lim_{t\to\infty}\sup h(t)$, $h^l = \lim_{t\to\infty}\inf h(t)$.

In system (1), we always assume that

(H$_1$) Functions $r_i(t), a_{ij}(t)$ $(i,j = 1,2,3)$ and $D_{ij}(t)$ $(i,j=1,2, i \neq j)$ are positive and continuous for all $t \in R$.

(H$_2$) Functions $k_i(t,s)$ $(i=1,2,3)$ are defined on $R \times [-\varrho_i, 0]$ and integrable with respect to s on $[-\varrho_i, 0]$ such that $\int_{-\varrho_i}^{0} k_i(t,s)\,\mathrm{d}s$ $(i=1,2,3)$ are positive, continuous and bounded with respect to t on R. There are nonnegative and continuous functions $h_i(s)$ defined on $[-\varrho_i, 0]$ satisfying $0 < \int_{-\varrho_i}^{0} (-s)h_i(s)\,\mathrm{d}s < \infty$ such that $k_i(t,s) \leqslant h_i(s)$ for all t on R, s on $[-\varrho_i, 0]$ and $i = 1,2,3$.