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The variational iteration method for solving two forms of Blasius equation on a half-infinite domain

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Abstract

The variational iteration method is applied for a reliable treatment of two forms of the third order nonlinear Blasius equation which comes from boundary layer equations. The study shows that the series solution is obtained without restrictions on the nonlinearity behavior. The obtained series solution is combined with the diagonal Padé approximants to handle the boundary condition at infinity for only one of these forms.

Keywords: The variational iteration method; Boundary layers; Blasius equation; Padé approximants

1. Introduction

Blasius equation is one of the basic equations of fluid dynamics. Blasius equation describes the velocity profile of the fluid in the boundary layer theory [1,2] on a half-infinite interval. A broad class of analytical solutions methods and numerical solutions methods in [1–5] were used to handle this problem.

Two forms of Blasius equation appear in the fluid mechanic theory, where each is subjected to specific physical conditions. The equation has the forms

$$\begin{aligned} &u'''(x) + \frac{1}{2}u(x)u''(x) = 0, \\ &u(0) = 0, \quad u'(0) = 1, \quad u'(\infty) = 0, \end{aligned} \tag{1}$$

and

$$\begin{aligned} &u'''(x) + \frac{1}{2}u(x)u''(x) = 0, \\ &u(0) = 0, \quad u'(0) = 0, \quad u'(\infty) = 1. \end{aligned} \tag{2}$$

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It is obvious that the differential equations are the same, but differ in boundary conditions. For more details about the appearance of the two forms, see reference [1]. It is well known that the Blasius equation is the mother of all boundary-layer equations in fluid mechanics. Many different, but related, equations have been derived for a multitude of fluid-mechanical situations, for instance, the Falkner–Skan equation [1].

An analytic treatment will be approached to find the numerical values of $u''(0)$ for both boundary value problems. The goal will be achieved by using the reliable variational iteration method developed by He in [5–10] and used in [11–14] and the references therein. The obtained series is best manipulated by Padé approximants for numerical approximations [15–19] for the first form (1). Using the power series, isolated from other concepts, is not always useful because the radius of convergence of the series may not contain the two boundaries.

The variational iteration method (VIM) established in (1999) by He in [5–10] is thoroughly used by many researchers to handle linear and nonlinear models. The method has been used by many authors in [11–14] and the references therein to handle a wide variety of scientific and engineering applications: linear and nonlinear, and homogeneous and inhomogeneous as well. It was shown by many authors that this method provides improvements over existing numerical techniques. The method gives successive approximations of high accuracy of the solution. The VIM does not require specific treatments as in Adomian method, and perturbation techniques for nonlinear terms.

In this paper, only a brief discussion of the variational iteration method will be emphasized, because the complete details of the method are found in [5–14] and in many related works. The objectives of this work are twofold. Firstly, we aim to apply the variational iteration method (VIM) in a direct manner to establish series solutions for Eqs. (1) and (2). Secondly, we seek to show the power of the method in handling linear and nonlinear equations in a unified manner without requiring any additional restriction. The VIM method is capable of greatly reducing the size of calculations while still maintaining high accuracy of the numerical solution. In what follows, we highlight the main steps of the He's variational iteration method.

2. The He's variational iteration method

Consider the differential equation

$$Lu + Nu = g(x,t), \tag{3}$$

where L and N are linear and nonlinear operators respectively, and $g(x,t)$ is the source inhomogeneous term. In [5–10], the variational iteration method was proposed by He where a correction functional for Eq. (3) can be written as

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda(Lu_n(\xi) + N\tilde{u}_n(\xi) - g(\xi))\,\mathrm{d}\xi, \quad n \geqslant 0. \tag{4}$$

It is obvious that the successive approximations u_j, $j \geqslant 0$ can be established by determining λ, a general Lagrange's multiplier, which can be identified optimally via the variational theory. The function $\tilde{u}_n$ is a restricted variation which means $\delta\tilde{u}_n = 0$. Therefore, we first determine the Lagrange multiplier λ that will be identified optimally via integration by parts. The successive approximations $u_{n+1}(x,t)$, $n \geqslant 0$ of the solution $u(x,t)$ will be readily obtained upon using the obtained Lagrange multiplier and by using any selective function u_0. The initial values $u(x,0)$ and $u_t(x,0)$ are usually used for selecting the zeroth approximation u_0. With λ determined, then several approximations $u_j(x,t)$, $j \geqslant 0$, follow immediately. Consequently, the exact solution may be obtained by using

$$u = \lim_{n\to\infty} u_n. \tag{5}$$

In what follows, we will apply the VIM method for the Blasius forms (1) and (2) to illustrate the strength of the method and to establish approximations of high accuracy for these models.