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Non-linear mathematical models for blood flow through tapered tubes

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Abstract

In this paper, the steady flow of blood through tapered tube has been analyzed assuming blood as (i) Casson fluid and (ii) Herschel–Bulkley fluid. The expressions for pressure drop, wall shear stress and resistance to flow have been obtained. The effects of tapering of the tube and the non-Newtonian nature of the fluid on pressure drop, wall shear stress and resistance to flow are discussed. For all fluids, the pressure drop increases with increasing angle of taper from 0.5° to 1° for a given value of yield stress θ and tapered tube Reynolds number Re_ψ. The resistance to flow as well as the wall shear stress increase with increasing yield stress for Herschel–Bulkley fluid and also for Casson's fluid when the other parameters held constant. Both for Herschel–Bulkley fluid and Casson's fluid, the wall shear stress as well as the resistance to flow increase with increasing axial distance for a given tapered tube Reynolds number Re_ψ, angle of taper ψ and yield stress θ.

Keywords: Steady flow; Tapered tube; Herschel–Bulkley fluid; Casson fluid; Pressure drop; Resistance to flow

1. Introduction

The analysis of blood flow through tapered tubes is very important in understanding the behaviour of the flow as the taper of the tube is an important factor in the pressure development. It has been pointed out that the blood vessels bifurcate at frequent intervals and although the individual segments of arteries may be treated as uniform between bifurcations, the diameter of the artery decreases quite fast at each bifurcation [1]. It has been observed that even for the small angles of taper (upto 2°), the effects of tapering of the blood vessels cannot be neglected [2]. As pointed out by How and Black [3], this study is also very useful for the design of prosthetic blood vessels as the use of grafts of tapered lumen has the advantage of surgical benefits, the blood vessels being wider upstream.

The important hydrodynamical factor for tapered tube geometry is the pressure loss which leads to the diminished blood flow through the grafts. Several authors have reported theoretical and experimental study

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Nomenclature

$\overline{K}$	coefficient of viscosity for non-Newtonian fluid
$\overline{L}_1$	axial distance of the cross section $\bar{z} = 0$ from cone apex
$\overline{L}_2$	axial distance of any cross section from cone apex
n	power law index
$\bar{p}$	pressure
p	dimensionless pressure
$\bar{r}$	radial distance
r	dimensionless radial distance
$\overline{Q}$	flow rate
Re_ψ	tapered tube Reynolds number
$\overline{R}$	radius of the tapered tube
$\overline{R}_0$	radius of the normal artery
$\overline{U}_0$	typical velocity
$\bar{u}$	axial velocity
$\bar{z}$	axial distance
z	dimensionless axial distance

Greek letters

$\Delta\bar{p}$	pressure drop
ΔP	dimensionless pressure drop
$f(\bar{\tau})$	flow curve for non-Newtonian fluid
$\overline{\Lambda}$	resistance to flow
Λ	dimensionless resistance to flow
θ	dimensionless yield stress
$\bar{\tau}$	shear stress
τ	dimensionless shear stress
$\bar{\tau}_{\mathrm{B}}$	yield stress for Bingham fluid
$\bar{\tau}_{\mathrm{C}}$	yield stress for Casson fluid
$\bar{\tau}_{\mathrm{H}}$	yield stress for Herschel–Bulkley fluid
$\bar{\tau}_{\mathrm{y}}$	yield stress
$\bar{\tau}_{\mathrm{w}}$	wall shear stress
τ_{w}	dimensionless wall shear stress
$\bar{\mu}_{\mathrm{B}}$	coefficient of viscosity for Bingham fluid
$\bar{\mu}_{\mathrm{C}}$	coefficient of viscosity for Casson fluid
$\bar{\mu}_{\mathrm{H}}$	coefficient of viscosity for Herschel–Bulkley fluid
$\bar{\mu}_{\mathrm{N}}$	coefficient of viscosity for Newtonian fluid
$\bar{\mu}_{\mathrm{p}}$	coefficient of viscosity for Power law fluid
ψ	angle of taper
ρ	density

Subscript

w	wall shear (used for τ)

of the pressure flow relationship for different fluids through tapered tubes such as Newtonian, Power law and Bingham [3–10]. However, it has been noticed in smaller blood vessels at low shear rates ($\dot{\gamma} < 10/\mathrm{s}$), the yield stress for blood is non-zero and blood behaves as a non-Newtonian fluid [11,12]. The non-Newtonian character of blood is typical in small arteries and veins where the presence of cells induce that specific behaviour [13]. It has been pointed out that in some diseased conditions e.g. patients with sever myocardial infarction,