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A probabilistic bi-level linear multi-objective programming problem to supply chain planning

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Abstract

Bi-level programming, a tool for modeling decentralized decisions, consists of the objective(s) of the leader at its first level and that is of the follower at the second level. Three level programming results when second level is itself a bi-level programming. By extending this idea it is possible to define multi-level programs with any number of levels.

In most of the real life problems in mathematical programming, the parameters are considered as random variables. The branch of mathematical programming which deals with the theory and methods for the solution of conditional extremum problems under incomplete information about the random parameters is called "stochastic programming".

Supply chain planning problems are concerned with synchronizing and optimizing multiple activities involved in the enterprise, from the start of the process, such as procurement of the raw materials, through a series of process operations, to the end, such as distribution of the final product to customers.

Enterprise-wide supply chain planning problems naturally exhibit a multi-level decision network structure, where for example, one level may correspond to a local plant control/scheduling/planning problem and another level to a corresponding plant-wide planning/network problem. Such a multi-level decision network structure can be mathematically represented by using "multi-level programming" principles.

In this paper, we consider a "probabilistic bi-level linear multi-objective programming problem" and its application in enterprise-wide supply chain planning problem where (1) market demand, (2) production capacity of each plant and (3) resource available to all plants for each product are random variables and the constraints may consist of joint probability distributions or not. This probabilistic model is first converted into an equivalent deterministic model in each level, to which fuzzy programming technique is applied to solve the multi-objective nonlinear programming problem to obtain a compromise solution.

Keywords: Bi-level programming; Multi-objective decision-making; Multi-level multi-objective decision-making; Fuzzy decision-approach; Stochastic programming; Supply chain management

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1. Introduction and literature review

1.1. Bi-level programming

A bi-level programming problem is formulated for a problem in which two decision-makers make decisions successively. For example, in a decentralized firm, top management makes a decision such as budget of the firm, and then each division determines a production plane in the full knowledge of the budget [1].

Research on multi-level mathematical programming to solve organizational planning and decision-making problems has been conducted widely. The research and application have concentrated mainly on bi-level programming [1]. In the BLP problem, each decision maker tries to optimize its own objective function(s) without considering the objective(s) of the other party, but the decision of each party affects the objective value(s) of the other party as well as the decision space.

The general formulation of a bi-level programming problem (BLPP) is [2]:

$$\begin{aligned} &\min_x \quad F(x,y) \\ &\text{s.t.} \quad G(x,y) \leqslant 0, \\ &\min_y \quad f(x,y) \\ &\text{s.t.} \quad g(x,y) \leqslant 0, \end{aligned} \tag{1}$$

where $x \in R^{n1}$ and $y \in R^{n2}$.The variables of problem are divided into two classes, namely the upper-level variables $x \in R^{n1}$ and the lower-level variables $y \in R^{n2}$. Similarly, the functions $F: R^{n1} \times R^{n2} \to R$ and $f: R^{n1} \times R^{n2} \to R$ are the upper-level and lower-level objective functions respectively, while the vector-valued functions $G: R^{n1} \times R^{n2} \to R^{m1}$ and $g: R^{n1} \times R^{n2} \to R^{m2}$ are called the upper-level and lower-level constraints respectively. All of the constraints and objective functions may be linear, quadratic, nonlinear, fractional, etc.

1.2. Stochastic programming

In most of the real life problems in mathematical programming, the parameters are considered as random variables. The branch of mathematical programming which deals with the theory and methods for the solution of conditional extremum problems under incomplete information about the random parameters is called "stochastic programming". Most of the problems in applied mathematics may be considered as belonging to any one of the following classes [7]:

1. Descriptive Problems, in which, with the help of mathematical methods, information is processed about the investigated event, some laws of the event being induced by others.
2. Optimization Problems in which from a set of feasible solutions, an optimal solution is chosen.

Besides the above division of applied mathematics problems, they may be further classified as deterministic and stochastic problems. In the process of the solution of the stochastic problem, several mathematical methods have been developed. However, probabilistic methods were for a long time applied exclusively to the solution of the descriptive type of problems. Research on the theoretical development of stochastic programming is going on for the last four decades. To the several real life problems in management science, it has been applied successfully [13]. The chance constrained programming was first developed by Charnes and Cooper [4]. Subsequently, some researchers like Sengupta [12], Contini [5], Sullivan and Fitzsimmons [14], Leclercq [9], Teghem et al. [15] and many others have established some theoretical results in the field of stochastic programming. Stancu-Minasian and Wets [13] have presented a review paper on stochastic programming with a single objective function.

The fuzziness occurs in many of the real life decision making problems. Decision making in a fuzzy environment was first developed by Bellman and Zadeh [3]. Zimmermann [16] presented an application of fuzzy linear programming to the linear vector-maximum problem and showed that the solution obtained by fuzzy