Available online at www.sciencedirect.com
ScienceDirect
Applied Mathematics and Computation 190 (2007) 231–236
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

An stable numerical algorithm for identifying the solution of an inverse problem

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Abstract

In this paper, we consider an inverse parabolic problem with space dependent coefficient. Mathematical model of the problem consists a parabolic equation in which the condition is unknown at the one of the boundary and to be determined from an overspecified data measured at an interior point inside the body. Uniqueness of the solution of under study inverse problem will be shown. Our concern for the numerical procedure for this inverse problem is based on a finite differences scheme. Stability conditions for numerical solution to inverse problem are stated. The approach of proposed method is approximated the unknown function by a set of Chebyshev polynomials, where the unknown set of expansion coefficients in unknown function are determined from the minimizing the least squares method. Some numerical examples will be given in the last section.

Keywords: Inverse parabolic problem; Stability; Minimization; Chebyshev polynomials; Least-squares method; Finite differences approximation

1. Introduction

Inverse problem of determination of unknown function in a parabolic differential equation has been treated by many authors [5,6,8–10]. The direct parabolic problem are concerned with determination of solution at interior points of the region when the initial and boundary conditions are specified [2]. In contrast, inverse problem involves the determination of the surface conditions from the knowledge at the interior point of the region. In many situation it is difficult to determine the solution analytically, such solution determined by employing inverse problem [1,3,4]. In this paper, we shall deal with the identification of solution at $x = 1$ in an inverse parabolic problem with space dependent coefficient. In fact, our aim is to find unknown function from a known solution at a fix interior point x_0 inside the body.

The structure of this paper is organized as follows. In the next section, mathematical model for this inverse problem is shown and uniqueness of the solution to this inverse problem will be proved. In Section 3 the

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0096-3003/$ - see front matter
doi:10.1016/j.amc.2007.01.022

numerical procedure based on finite differences scheme are described, and stability conditions for numerical solution will be shown. Finally, we find the unknown function in inverse problem from the solution of the minimizing least squares method. In Section 4 some numerical examples will be presented.

2. Mathematical model

In this section, we consider the following inverse problem:

$$u_t = (a(x)u_x)_x, \quad 0 < x < 1, \;\; t > 0, \tag{1}$$

$$u(x,0) = f(x), \quad 0 < x < 1, \tag{2}$$

$$u(0,t) = p(t), \quad t > 0, \tag{3}$$

$$u(1,t) = \phi(t), \quad t > 0, \tag{4}$$

where $f(x)$, $p(t)$, $a(x)$ are considered as known functions, $\phi(t)$ and $u(x,t)$ are unknown functions. In order to determine $u(1,t) = \phi(t)$, let us use an additional condition given at the interior point, $x = x_0$ of the region

$$u(x_0,t) = q(t), \quad 0 < x_0 < 1, \;\; t > 0. \tag{5}$$

The problem (1)–(5) can be divided into two separated problems, one of them is the following direct problem:

$$u_t = (a(x)u_x)_x, \quad 0 < x < x_0, \;\; t > 0, \tag{6}$$

$$u(x,0) = f(x), \quad 0 < x < x_0, \tag{7}$$

$$u(0,t) = p(t), \quad t > 0, \tag{8}$$

$$u(x_0,t) = q(t), \quad t > 0, \tag{9}$$

because there are known initial and boundary conditions. Another problem is the following inverse problem:

$$u_t = (a(x)u_x)_x, \quad x_0 < x < 1, \;\; t > 0, \tag{10}$$

$$u(x,0) = f(x), \quad x_0 < x < 1, \tag{11}$$

$$u(x_0,t) = q(t), \quad t > 0, \tag{12}$$

$$u(1,t) = \phi(t), \quad t > 0. \tag{13}$$

Throughout this paper, we assume that

$$a \in C^1[0,\infty), \quad 0 < a_0 \leqslant a(x) \leqslant A_0. \tag{14}$$

2.1. Uniqueness of the solution to inverse problem (1)–(5)

In order to prove the unicity of the solution to inverse problem (1)–(5), let us suppose that u_1 and u_2 are two solutions of inverse problem. By putting $u = u_1 - u_2$, It can be shown that

$$u(x,t) = 0, \quad 0 \leqslant x \leqslant x_0, \;\; t > 0.$$

Therefore,

$$u(x_0,t) = u_x(x_0,t) = 0, \quad t > 0,$$

and,

$$u_t = (a(x)u_x)_x, \quad x_0 \leqslant x \leqslant 1, \;\; t > 0,$$
$$u(x_0,t) = u_x(x_0,t) = 0, \quad t > 0.$$

Applying Laplace transformation to previous problem yields

$$s\overline{U} - (a(x)\overline{U}_x)_x = 0,$$
$$\overline{U}(x_0,t) = 0, \quad \overline{U}_x(x_0,t) = 0,$$