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The tanh method and Adomian decomposition method for solving the foam drainage equation

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Abstract

Foaming occurs in many distillation and absorption processes. The drainage of liquid foams involves the interplay of gravity, surface tension, and viscous forces. In this paper, we use a semi analytic method, the Adomian decomposition method, and an analytic method, the tanh method to handle the foam drainage equation. The powerful tanh method gives the solution in a closed form. However, Adomian decomposition method computes the solution in a rapidly convergent infinite series. The comparison between the two approaches is conducted to illustrate the performance of each method.

Keywords: Foam drainage equation; Decomposition method; The tanh method

1. Introduction

Foams are of great importance in many technological processes and applications, and their properties are subject of intensive studies from both practical and scientific points of view [1]. Liquid foam is an example of soft matter (or a complex fluid) with a very well-defined structure, first clearly described by Joseph Plateau in the 19th century. Weaire [2] had mentioned in his research work many aspects of the fluid dynamics of this system, that can be summarized as follows:

1. How is liquid transported through it in response to a pressure gradient or gravity?
2. How does it respond to stress, particularly above the yield stress?
3. What is the nature of the local fluid flow in the Plateau borders and their junctions?

Weaire [2] showed in his work that simple first-order answers to many such questions exist but on going experiments continue to challenge our understanding.

Foams and emulsions are well known to scientists and the general public alike because of their everyday occurrence [3,4]. Foams are common in foods and personal care products such as creams and lotions, and

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foams often occur, even when not desired, during cleaning (clothes, dishes, scrubbing) and dispensing processes (c.f. [5]). They have important applications in the food and chemical industries, firefighting, mineral processing, and structural material science (c.f. [6]).

Less obviously they appear in acoustic cladding, lightweight mechanical components, impact absorbing parts on cars, heat exchangers and textured wallpapers (incorporated as foaming inks) and even have an analogy in cosmology (the clustering of galaxies).

The packing of bubbles or cells can form both random and symmetrical arrays, such as sea foam and bees' honeycomb.

History connects foams with a number of eminent scientists, and foams continue to excite imaginations [7].

Although there are now many applications of polymeric foams [8] and more recently metallic foams, which are foams made out of metals such as aluminum [9]. Some commonly mentioned applications include the use of foams for reducing the impact of explosions and for cleaning up oil spills.

In addition, industrial applications of polymeric foams and porous metals include their use for structural purposes (e.g., lightweight sandwich structures) and as heat exchange media analogous to common ''finned'' structures [10].

Polymeric foams are used in cushions and packing and structural materials [11]. Glass, ceramic, and metal foams [12] can also be made, and find an increasing number of new applications. In addition, mineral processing utilizes foam to separate valuable products by flotation. Finally, foams enter geophysical studies of the mechanics of volcanic eruptions [5].

Recent research in foams and emulsions has centered on three topics which are often treated separately, but are in fact interdependent: drainage, coarsening, and rheology; see Fig. 1. We focus here on a quantitative description of the coupling of drainage and coarsening.

Foam drainage is the flow of liquid through channels (Plateau borders) and nodes (intersections of four channels) between the bubbles, driven by gravity and capillarity [13–15] (see [6]). During foam production the material is in the liquid state and fluid can rearrange while the bubble structure stays relatively unchanged. The flow of liquid relative to the bubbles is called drainage. Generally, drainage is driven by gravity and/or capillary (surface tension) forces and is resisted by viscous forces (c.f. [5]).

Because of their limited time stability and despite the numerous studies reported in the literature, many of their properties are still not well understood, in particular the drainage of the liquid in between the bubbles under the influence of gravity [16,17]. Drainage plays an important role in foam stability: indeed, when a foam dries, its structure becomes more fragile; the liquid films between adjacent bubbles being thinner, then can break, leading to foam collapse. In the case of aqueous foams, surfactant is added into water and it adsorbs at the surface of the films, protecting them against rupture (c.f. [18]).

Most of the basic rules that explain the stability of liquid gas foams were introduced over 100 years ago by the Belgian, Joseph Plateau, who was blind before he completed his important book on the subject. This modern-day book by Weaire and Hutzler provides valuable summaries of Plateau's work on the laws of equilib-

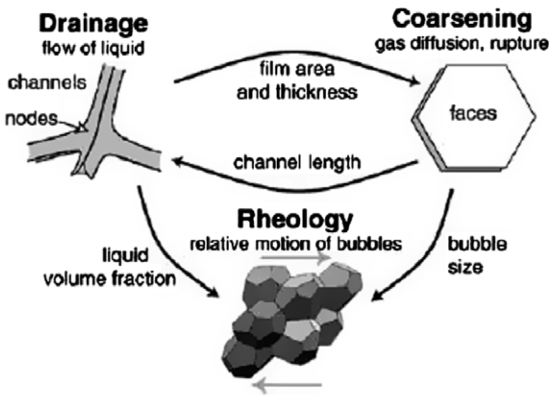

Fig. 1. Hilgenfeldt [6].