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Numerical solution of a system of fourth order boundary value problems using cubic non-polynomial spline method

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Abstract

A cubic non-polynomial spline technique is developed for the numerical solutions of a system of fourth order boundary value problems associated with obstacle, unilateral and contact problems. The end conditions consistent with the BVP are derived corresponding to the boundary conditions in terms of not only second derivatives but first derivatives as well. The present method has less computational cost and gives better approximations than those produced by other collocation, finite difference and spline methods. The method developed is compared with those developed by Khan et al. [A. Khan, M.A. Noor, T. Aziz, Parametric quintic-spline approach to the solution of a system of fourth-order boundary-value problems, Journal of Optimization Theory and Applications 122(2) (2004) 309–322], Al-Said and Noor [E.A. Al-Said, M.A. Noor, Computational methods for fourth-order obstacle boundary-value problems, Communications in Applied Nonlinear Analysis, 2 (1995) 73–83] and Siddiqi and Ghazala [S.S. Siddiqi, G. Akram, Solution of the system of fourth-order boundary-value problems using non polynomial spline technique, Applied Mathematics and Computation 185 (2007) 128–135] through different examples.

Keywords: Non-polynomial spline; System of boundary value problems; Obstacle problems; Variational inequalities

1. Introduction

Variational inequality theory has become an effective and powerful tool for studying the contact, unilateral, obstacle and equilibrium problems arising in different branches of pure and applied sciences. Variational inequality theory has proved to be immensely useful in the study of many branches of mathematical and engineering sciences. Penalty methods and projection methods have been developed for the solution of general variational inequalities (see [1–4,6,8–10,12,14,19]). Since in projection methods, the projection is needed, which is difficult to be obtained so the projection methods are not supposed to be suitable. The penalty methods are inefficient as in these methods, instability is created. However, the general variational inequalities can be characterized by a system of differential equations using the penalty function technique, if the obstacle function is known. This technique was used by Lewy and Stampacchia [8] to study the regularity of the solution of

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variational inequalities. The main advantage of this technique is its simple applicability in solving obstacle and unilateral problems. This technique has been used for solving fourth order system of differential equations associated with obstacle and unilateral problems by finite difference and quintic spline methods [1,2,19].

Noor and Tirmizi [10] solved the system of second order boundary value problems using pade approximation. Al-Said [4] developed the method for the solution of system of second order boundary value problems using cubic spline. It is claimed that the method can be considered as an improvement for the cubic spline method developed in [4]. Gao and Chi [6] solved a system of third-order boundary value problems associated with third-order obstacle problems using the quartic B-splines and the method is claimed to be of second order. Siraj et al. [20] developed the solution of a system of third-order boundary value problems using non polynomial spline and the method is claimed to be of second order as well.

Usmani [15], developed the method of the solution of fourth order boundary value problem, considering it to be the problem of bending a rectangular clamped beam of length l resting on an elastic foundation. The vertical deflection w of the beam satisfies the system

$$\left.\begin{aligned}&\left[L+\left(\tfrac{K}{D}\right)\right]w = D^{-1}q(x),\\ &w(0)=w(l)=w'(0)=w'(l)=0,\end{aligned}\right\} \tag{1.1}$$

where $L \equiv \frac{d^4}{dx^4}$, D is the flexural rigidity of the beam, K is the spring constant of the elastic foundation and the load $q(x)$ acts vertically downwards per unit length of the beam.

Usmani [16], developed the discrete methods for the solution of fourth order linear special case boundary value problem, similar to the problem (1.1) with the change in boundary conditions in terms of second order derivatives instead of first order derivatives. Twizell and Tirmizi [5] developed and analysed a sixth-order method for the numerical solution of the linear fourth-order boundary value problem for which the boundary conditions are given in terms of functional values and (i) first-order derivatives (the clamped–clamped beam problem) or (ii) second-order derivatives (the simple–simple beam problem). Papamichael and Worsey [13] developed the cubic spline method for the solution of problem (1.1). Daele et al. [11] developed second order method for the solution of fourth-order boundary value problem using non polynomial spline which is mixture of mixed spline function consisting of cubic polynomial function and a trigonometric function. Siddiqi and Ghazala [17,18] developed the non polynomial spline technique for the solution of linear special case fifth- and eighth-order boundary value problems, respectively. Ghazala and Siddiqi [7] solved the sixth order linear special case boundary value problems using non polynomial spline. Siddiqi and Ghazala [19] solved a system of fourth-order boundary value problems associated with obstacle problems using quintic non polynomial spline.

The following system of fourth order boundary value problems, is considered

$$y^{(4)}(x) = \begin{cases} f(x), & a \leqslant x \leqslant c,\\ f(x)+y(x)g(x)+r, & c \leqslant x \leqslant d,\\ f(x), & d \leqslant x \leqslant b,\end{cases} \tag{1.2}$$

along with the following two cases of boundary conditions

Case I

$$\left.\begin{aligned}&y(a)=y(b)=\alpha_0, \quad y'(a)=y'(b)=\alpha_1,\\ &y(c)=y(d)=\alpha_2, \quad y\prime(c)=y'(d)=\alpha_3,\end{aligned}\right\} \tag{1.3}$$

Case II

$$\left.\begin{aligned}&y(a)=y(b)=\alpha_0, \quad y''(a)=y''(b)=\alpha_4,\\ &y(c)=y(d)=\alpha_2, \quad y''(c)=y''(d)=\alpha_5,\end{aligned}\right\} \tag{1.4}$$

where r and $\alpha_i, i=0,1,\ldots,5$ are finite real constants and the functions $f(x)$ and $g(x)$ are continuous on $[a,b]$ and $[c,d]$, respectively. Such type of systems arise in connection with contact, obstacle and unilateral problems.