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New complexiton solutions of the nonlinear evolution equations using a generalized rational expansion method with symbolic computation ☆

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Abstract

In this paper, we study complexiton solutions of nonlinear evolution equations by using a generalized ansätz. With the help of symbolic computation *Maple*, we obtain new types of complexiton solutions of the $(2+1)$-dimensional Burgers equation. The solutions contain the combination of hyperbolic function and elliptic function, trigonometric function and elliptic function.

Keywords: Nonlinear evolution equations; Generalized ansätz; Complexiton solutions; $(2+1)$-dimensional Burgers equation

1. Introduction

Recently, directly searching for exact solutions of nonlinear evolution equations (NEEs) has become more and more attractive, on the one hand, due to their occurrence in many fields of science, in physics as well as in chemistry or biology and the interesting features and rich variety of their solutions, on the other hand, due to the availability of computer systems like Maple or Mathematica which allow us to perform some complicated and tedious algebraic calculation and differential calculation on a computer, at the same time help us to find new exact solution of NEEs. In order to obtain exact solutions of NEEs, many effective methods have been presented, such as, inverse scattering method [1], Bäcklund transformation [2,3], Darboux transformation [4], Hirota bilinear method [5], similarity reductions method [6,7], variable separation approach [8], Painlevé analysis method [9], homogeneous balance method [10], various tanh function methods [11–24] and so on. Among them, the tanh function method is considered to be one of the most straightforward and effective algebraic algorithm to obtain exact solutions for lots of NEEs. As we know, when applying the tanh function method, the choice of an appropriate sub-equation is of great importance. Much work has been concentrated on solving more solutions of the sub-equation. Here, we focus on the multiform choice of sub-equation.

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Recently, Ma [25] found a novel class of explicit exact solutions to the Korteweg–de Vries equation through its bilinear form and defined the solutions as complexiton solutions. In Ref. [26], Lou et al. presented and answered the problem "Are there any exact explicit multiple periodic wave solutions and periodic-solitary wave solutions for the nSG equation" with help of the mapping relations among the sine-Gordon field equation and the cubic nonlinear Klein–Gordon fields. Such solutions possess singularities of combinations of trigonometric function waves and exponential function waves which have different travelling speeds of new type. Above mentioned solutions of nonlinear evolution equations have a common character: combination of trigonometric function waves and exponential function waves. For unification and conciseness, so we call the solutions obtained by Ma and the solutions obtained by Lou as complexiton solutions.

To our knowledge, much work can not obtain such complexiton solutions [25] and [26]. In line with the development of computerized symbolic computation, the present work is motivated by the desire to present the generalized rational expansion method with the introduced variable satisfying the multiple sub-equation to obtain more new types of complexiton solutions. For illustration, we apply the generalized method to solve $(2+1)$-dimensional Burgers equation and successfully construct new and more general complexiton solutions.

This paper is organized as follows. In Section 2, our method is summarized. In Section 3, it is applied to $(2+1)$-dimensional Burgers equation and obtain some complexiton solutions for this model. A short summary and discussion are given in final.

2. Summary of our method

In the following we would like to outline the main steps of our method:

Step 1. Given a system of polynomial NEEs with some physical fields $u_i(x,y,t)$ in three variables x,y,t,

$$\mathcal{N}(u_i, u_{it}, u_{ix}, u_{iy}, u_{itt}, u_{ixt}, u_{iyt}, u_{ixx}, u_{iyy}, u_{ixy}, \ldots) = 0. \tag{2.1}$$

Step 2. We introduce a more generalized ansätz in terms of finite rational formal expansion in the following forms:

$$u_i(x,y,t) = \mathcal{R}_{u_i}(\phi_1(\xi_1), \phi_2(\xi_2), \ldots, \phi_n(\xi_n)), \tag{2.2}$$

where $\mathcal{R}_{u_i}(\cdot)$ is the rational formal function of $\phi_1(\xi_1), \phi_2(\xi_2), \ldots, \phi_n(\xi_n)$, $\xi_1 = k_1(x + l_1 y - \lambda_1 t)$, $\xi_2 = k_2(x + l_2 y - \lambda_2 t)$, $\ldots$, $\xi_n = k_n(x + l_n y - \lambda_n t)$, and $k_1, l_1, \lambda_1, k_2, l_2, \lambda_2, \ldots, \lambda_n$ are arbitrary constants. And $\phi_1(\xi_1), \phi_2(\xi_2), \ldots, \phi_n(\xi_n)$ satisfy:

(1) $\phi_1(\xi_1), \phi_2(\xi_2), \ldots, \phi_n(\xi_n)$ are arbitrary function;

(2) $\phi_1(\xi_1), \phi_2(\xi_2), \ldots, \phi_n(\xi_n)$ satisfy various sub-equation, such as elliptic equation, Riccati equation, projective Riccati equation, and so on. That is, all derivatives of $\phi_i(\xi_i)$ $(i = 1, \ldots, n)$ with respect to ξ_i are the rational formal function of $\phi_1(\xi_1), \phi_2(\xi_2), \ldots, \phi_n(\xi_n)$.

For example, we can take $n = 1$,

$$u_i(x,y,t) = a_{i0} + \sum_{j=1}^{m_i} \frac{a_{ij}(\phi_1(\xi_1))^j}{(\mu_0 + \mu_1\phi_1(\xi_1))^j}, \tag{2.3}$$

where $a_0,\ a_1,\ \mu_0$ and μ_1 are differentiable functions in three variables x, y, t, or arbitrary constants to be determined later.

We can take $n = 2$,

$$u_i(x,y,t) = a_{i0} + \sum_{j=1}^{m_i} \frac{\sum_{r_{j1}+r_{j2}=j} a^{ij}_{r_{j1}r_{j2}}(\phi_1(\xi_1))^{r_{j1}}(\phi_2(\xi_1))^{r_{j2}}}{(\mu_0 + \mu_1\phi_1(\xi_1) + \mu_2\phi_2(\xi_1) + \mu_3\phi_1(\xi_1)\phi_2(\xi_1))^j}, \tag{2.4}$$

$$u_i(x,y,t) = a_{i0} + \sum_{j=1}^{m_i} \frac{\sum_{r_{j1}+r_{j2}=j} a^{ij}_{r_{j1}r_{j2}}(\phi_1(\xi_1))^{r_{j1}}(\phi_2(\xi_2))^{r_{j2}}}{(\mu_0 + \mu_1\phi_1(\xi_1) + \mu_2\phi_2(\xi_2) + \mu_3\phi_1(\xi_1)\phi_2(\xi_2))^j}, \tag{2.5}$$

where $a_0,\ a_1,\ a_2,\ \mu_0,\ \mu_1,\ \mu_2$ and μ_3 are differentiable functions in three variables x, y, t, or arbitrary constants to be determined later.