Available online at www.sciencedirect.com
ScienceDirect
Applied Mathematics and Computation 189 (2007) 366–370
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

A family of modified super-Halley methods with fourth-order convergence

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Abstract

In this paper, we present a family of modified super-Halley methods for solving non-linear equations. Analysis of convergence shows that the methods have fourth-order convergence. The superiority of the new methods is that they require no additional evaluations of the function, the first derivative or second derivative as compared with the classical third-order methods although their order is improved. Numerical results show that the new methods can be efficient.
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Keywords: Super-Halley method; Newton's method; Non-linear equations; Iterative method

1. Introduction

Solving non-linear equations is one of the most important problems in numerical analysis. In this paper, we consider iterative methods to find a simple root of a non-linear equation $f(x) = 0$, where $f : D \subset \mathbb{R} \to \mathbb{R}$ for an open interval D is a scalar function.

Newton's method for a single non-linear equation is defined by:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}. \tag{1}$$

This is an important and basic method [1], which converges quadratically.

In order to accelerate Newton's method, many third-order methods are developed. A well-known third-order methods, called super-Halley method [2,3], is defined by

$$x_{n+1} = x_n - \left(1 + \frac{1}{2}\frac{L_f(x_n)}{1 - L_f(x_n)}\right)\frac{f(x_n)}{f'(x_n)}, \tag{2}$$

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0096-3003/$ - see front matter © 2006 Published by Elsevier Inc.
doi:10.1016/j.amc.2006.11.092

where

$$L_f(x_n)=\frac{f''(x_n)f(x_n)}{f'(x_n)^2}. \tag{3}$$

This method is, in general, an iterative process with order of convergence three although the method converges with fourth-order when it is applied to quadratic equations [4]. From a practical point of view, it is interesting and expected to research higher-order methods.

Recently, an approach is developed to modify super-Halley method by using the second derivative f'' at $(x_n-f(x_n))/(3f'(x_n))$ instead of x_n, and then a variant of super-Halley method with fourth-order convergence is obtained in [5]

$$x_{n+1}=x_n-\left(1+\frac{1}{2}\frac{K_f(x_n)}{1-K_f(x_n)}\right)\frac{f(x_n)}{f'(x_n)}, \tag{4}$$

where

$$K_f(x_n)=\frac{f''(x_n-f(x_n)/(3f'(x_n)))f(x_n)}{f'(x_n)^2}. \tag{5}$$

This method is very interesting because its local order of convergence can be improved although it requires the same evaluations of the function, the first derivative and second derivative as its classical predecessor, super-Halley method.

On the other hand, the super-Halley method can be viewed as a particular one of the following general form defined by:

$$x_{n+1}=x_n-\left(1+\frac{1}{2}L_f(x_n)+\frac{1}{2}\frac{L_f(x_n)^2}{1-\alpha L_f(x_n)}\right)\frac{f(x_n)}{f'(x_n)},\quad \alpha\in\mathbb{R}. \tag{6}$$

This family of third-order methods is the particular case of the methods presented in [6]. Obviously, this family includes the classical super-Halley method ($\alpha=1$).

In this paper, we apply the approach used in [5] to (6) and obtain a family of modified super-Halley methods with fourth-order convergence. A detailed convergence analysis of the new methods is supplied. The new methods require one evaluation of the function, one of its first derivative and one of its second derivative, which are the same as super-Halley method. Consequently, the new methods could be of practical interest, as we show in some examples.

2. The methods and analysis of convergence

Now, we use $K_f(x_n)$ in (6) instead of $L_f(x_n)$ and obtain a family of new methods

$$x_{n+1}=x_n-\left(1+\frac{1}{2}K_f(x_n)+\frac{1}{2}\frac{K_f(x_n)^2}{1-\alpha K_f(x_n)}\right)\frac{f(x_n)}{f'(x_n)}, \tag{7}$$

where $K_f(x_n)$ is defined by (5). Obviously, when we take $\theta=1$, the method defined by (4) is obtained. Per iteration the new methods require no additional evaluations of the function, the first derivative or second derivative as compared with the methods defined by (6), but the order of convergence can be improved, and indeed, for (7), we have

Theorem 1. *Assume that the function $f: D\subset\mathbb{R}\to\mathbb{R}$ for an open interval D has a simple root $x^*\in D$. Let $f(x)$ be sufficiently smooth in the neighborhood of the root x^*, then the order of convergence of the methods defined by (7) is four.*

Proof. Let $e_n=x_n-x^*$ and $d_n=y_n-x^*$, where $y_n=x_n-f(x_n)/(3f'(x_n))$. Using Taylor expansion and taking into account $f(x^*)=0$, we have

$$f(x_n)=f'(x^*)[e_n+c_2e_n^2+c_3e_n^3+c_4e_n^4+\mathrm{O}(e_n^5)], \tag{8}$$

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