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On the solutions of tridiagonal linear systems

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Abstract

Tridiagonal linear systems are of special importance being appeared in space researches and also in several well-known algorithms. There are many methods for solving a tridiagonal linear system. Most of them are suffering from the accumulation of the round off errors. In this paper a new method will be given to overcome the defects of the other methods. Also, two direct methods, for solving a system of linear equations having matrices of quaternion entries as coefficients and independent terms, are proposed. Second kind Chebyshev polynomials, in a matrix argument, are used as a tool to do certain calculations.

Keywords: Tridiagonal linear system; Inverse matrix; Chebyshev polynomials

1. Introduction

A standard solution of a tridiagonal linear system

$$AX = B$$

can be obtained if we know the inverse matrix of A [11], which would reduce the problem of determining the solution for this system to one of matrix multiplication having the explicit form

$$X = A^{-1}B.$$

There are many methods for finding the inverse of a matrix such as the VU decomposition method and the direct inversion solution [5,9,12]. If the matrix is of small size, one of these methods will give the inverse without too much trouble. However, for larger matrices and for matrices whose elements are numbers that are approximations, these techniques are not too satisfactory [14]. As a result, a large number of numerical approximation methods for determining the inverse of a matrix have been developed; see for example, the forward and backward solutions of Croute method [5,14].

In the following method, we expect that the matrix A is no larger than 6×6. Therefore, a direct attack to get its inverse is not difficult. Therefore, one may use Gauss–Jordan elimination method for getting the inversion of the matrix A. However, this method is suffering from the accumulation of the round off errors being

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appeared due to the several divisions, subtractions and addition during the performance of the inversion. The decomposition method is therefore, may be better. The tridiagonal matrix A, is considered in the treatise work of Burgoyne [6], for treating matrices for computational purposes. This method is an attractive direct attack procedure, which overcomes the defects of Gauss–Jordan elimination method. The following solution gives the same answer like the other solution but it is faster than any of them.

2. Solution of tridiagonal system

Now, consider a system of linear equations in the form

$$x_{\ell-1} + \lambda x_\ell + x_{\ell+1} = B_\ell, \quad \ell = 2, 3, \ldots, n, \quad x_1 = x_{n+1} = 0. \tag{1}$$

Here, the common notation that any square determinant may be written as "$\Delta_i = \Delta_{ii}$" having (i) rows and (i) columns is used. Thus referring to the matrix form

$$AX = B,$$

$$\begin{bmatrix} \lambda & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 1 & \lambda & 1 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 1 & \lambda & 1 & \cdots & 0 & 0 & 0 & 0 \\ . & . & . & . & \cdots & . & . & . & . \\ . & . & . & . & \cdots & . & . & . & . \\ 0 & 0 & 0 & 0 & \cdots & 1 & \lambda & 1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 & \lambda & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 & \lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ . \\ . \\ . \\ x_{n-1} \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ . \\ . \\ . \\ . \\ B_{n-1} \end{bmatrix}. \tag{2}$$

Its determinant can be written as

$$\Delta_{n-1} = \begin{vmatrix} \lambda & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 1 & \lambda & 1 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 1 & \lambda & 1 & \cdots & 0 & 0 & 0 & 0 \\ . & . & . & . & \cdots & . & . & . & . \\ . & . & . & . & \cdots & . & . & . & . \\ 0 & 0 & 0 & 0 & \cdots & 1 & \lambda & 1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 & \lambda & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 & \lambda \end{vmatrix}, \tag{3}$$

which has the following recurrence relations:

$$\begin{aligned} &\Delta_1 = \lambda\Delta_0 - \Delta_{-1}, \\ &\Delta_2 = \lambda\Delta_1 - \Delta_0, \\ &\Delta_3 = \lambda\Delta_2 - \Delta_1, \\ &\ldots \\ &\Delta_n = \lambda\Delta_{n-1} - \Delta_{n-2}, \quad n = 1, 2, \ldots, \end{aligned} \tag{4}$$

where

$$\Delta_{-1} = 0, \quad \Delta_0 = 1. \tag{5}$$

Formulae (4) and (5) can be used for determining the values of the determinants $\Delta_1, \Delta_2, \ldots, \Delta_{n-1}$.

On the other hand, by using matrix algebra we are able to put the minor $\Delta_n^{(\ell,k)}$ of the element in the ℓ-row and k-column in the determinant Δ_n as a product of two determinants of lower orders in the form