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A class of smoothing methods for mathematical programs with complementarity constraints

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Abstract

Mathematical programs with complementarity constraints (MPCC) is an important subclass of MPEC, and for conventional MPEC, we can transform it into the MPCC form in some manner. It is a nature way to solve MPCC by constructing a suitable approximation of the primal problem. In this paper, we present a class of smoothing methods for MPCC, it is a broader approximation, and by selecting an available probability density function, we can obtain a corresponding approximation of MPCC. We show that the linear independence constraint qualification holds for the class of smooth methods under some conditions. We also analyze the convergence properties of the accumulated point gotten by the class of smooth methods.

Keywords: Mathematical program with complementarity constrains (MPCC); C-stationarity; M-stationarity; B-stationarity

1. Introduction

In this paper, we consider the following mathematical programs with complementarity constraints (MPCC):

$$\begin{cases} \min & f(z) \\ \text{s.t.} & g(z) \leqslant 0;\ h(z) = 0, \\ & G(z) \geqslant 0;\ H(z) \geqslant 0, \\ & G(z)^{\mathrm{T}} H(z) = 0, \end{cases} \tag{1.1}$$

where f: $R^n \to R$, g: $R^n \to R^p$, h: $R^n \to R^q$, and G, H: $R^n \to R^m$ are all twice continuously differentiable functions. It is an important subclass of mathematical programs with equilibrium constraints (MPEC).

The major cause of difficulty in solving problem (1.1) is that standard constraint qualification at any feasible point (such as Mangasarian–Fromovitz constraints qualification (MFCQ) or the linear independence constraint qualification (LICQ)) never holds at any feasible points [7,8]. Hence standard methods are not guaranteed to solve such problem. On the other hand, MPCC appears in many practical fields such as engineering and economics, see [1] and reference therein. So it has been received increasing attention recently.

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It is a natural way to find some suitable approximations of an MPCC such that the problem can be solved by solving a sequence of subproblems. In [2,3], the authors used the perturbed Fischer–Burmeister to approximate the complementarity constraints in (1.1). By this way , the problem (1.1) is reformulated as a standard mathematical programming, and we can solve the primal problem by standard available method or software. In [3], Fukushima and Pang also analyze the property of the limited point of the iteration under the MPEC linear independence constraint qualification and an additional condition called asymptotic weak non-degeneracy, an accumulation point of KKT points satisfying the second order necessary conditions for their perturbed problems is a B-stationary point of the original problem.

In this paper, we are interested in developing a class of smoothing methods to solve problem (1.1) which contains the method in [2,3] as its special case. We will show that the linear independence constraint qualification (LICQ) holds for the class of the smooth methods under some conditions. We also analyze the properties of the accumulated point of the class of the smooth methods and show that it shares the similar convergence properties in [3] under some conditions.

The rest of the paper is organized as follows: in Section 2, we present the class smoothing method for (1.1). In Section 3, we give the analysis of the convergence property.

A brief note on notation to be used in this paper. The i-th component of G will be denoted G_i in the rest of the paper and similarly to the vector-valued functions, $\mathscr{F}$ denote the feasible set of problem (1.1) and for a function F: $R^n \to R^m$ and give vector $z \in R^n$, let

$$\mathscr{I}_F(z) = \{i : F_i = 0\}$$

stand for the active index set of F at z.

2. A class of smooth approximations of MPCC

It is known that the complementarity problem has the following equivalence:

$$a \geqslant 0,\ b \geqslant 0,\ ab = 0 \Longleftrightarrow a = (a-b)_+. \tag{2.1}$$

Therefore, if we have the smoothing approximation of the plus function then we can obtain the smooth approximation of MPCC at the mean time.

Chen and Mangasarian [4] introduced a class of smoothing functions that approximates the plus function x_+ by twice integrating a parameterized probability density function. More specifically, their smoothing function is defined as

$$\widehat{p}(x,\beta) = \int_{-\infty}^{x} \widehat{s}(t,\beta)\mathrm{d}t \tag{2.2}$$

and

$$\widehat{s}(x,\beta) = \int_{-\infty}^{x} \widehat{t}(t,\beta)\mathrm{d}t = \int_{-\infty}^{x} \frac{1}{\beta}\mathrm{d}\left(\frac{x}{\beta}\right)\mathrm{d}t, \tag{2.3}$$

where $0 \leqslant \beta \leqslant \infty$ is a positive parameter, $\mathrm{d}(x)$ is a probability density function that satisfies

$$\mathrm{d}(x) \geqslant 0 \quad \text{and} \quad \int_{-\infty}^{\infty} \mathrm{d}(x)\mathrm{d}x = 1. \tag{2.4}$$

Clearly, when β goes to *0*, the probability density function $\widehat{t}(x,\beta)$ approaches the Dirac delta function $\delta(x)$, where $\delta(x)$ is the function satisfies the following properties:

$$\delta(x) = \begin{cases} 0 & x \neq 0, \\ \infty & x = 0, \end{cases}$$

with $\int_{t1}^{t2} \delta(x)\mathrm{d}x = 1$ if $0 \in [t1, t2]$ (and zero otherwise). And the smoothing function $\widehat{p}(x,\beta)$ trends to the plus function x_+.

Now we give the properties of $\widehat{p}(x,\beta)$ as follows.