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DEA efficiency analysis: Efficient and anti-efficient frontier

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Abstract

Data envelopment analysis (DEA) is a methodology for identifying the efficient frontier of production possibility set. Using this efficient frontier, an efficiency score is derived to each decision making units. This study, proposes an alternative efficiency measure by using efficient and anti-efficient frontiers. Numerical experiments show the validity of the proposed efficiency measure and its compatibility with other measures of efficiency. The paper addresses the super-efficiency issue by using this measure.

Keywords: Data envelopment analysis; Efficiency; Super-efficiency

1. Introduction

Data envelopment analysis (DEA) is a mathematical programming method for evaluating the relative efficiency of decision making unit (DMU) with multiple inputs and outputs. The performance of a DMU depends only on the identified efficient frontier characterized by the DMUs with an unity efficiency score. The efficiency of a DMU is a scalar measure ranging between zero and one. This scalar value is measured through a linear programming model (see [3,2]). Specifically, the Charnes, Cooper and Rhodes (CCR) model deals with a radial measure. This radial measure is calculated from optimistic viewpoint for each DMU, because, it is assumed as the radial measure that deals with input excess. Entani et al. [4] considered the DEA efficiencies from both the optimistic and the pessimistic viewpoints. In their study, the obtained efficiency measures constructed an efficiency interval that the lower bound of this interval is the efficiency in pessimistic viewpoint and the upper bound one is the efficiency in optimistic viewpoint. Those author are not explicitly interested in obtaining measures of efficiency and super-efficiency by considering both efficient and anti-efficient frontiers. In this paper, we introduce two sets: production possibility set (PPS) and quasi-production possibility set (QPPS). We call the frontier of the PPS as efficient frontier and the frontier of QPPS as anti-efficient frontier. Using these two frontiers (efficient and anti-efficient frontiers) we define an alternative efficiency measure. It is shown that this efficiency measure is units invariant. Then, we used our method to determine a super-efficiency score. The approach is applied to 50 bank branches. The paper unfolds as follows: The following section provides production possibility set and quasi-production possibility set. An alternative efficiency measure is

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introduced in Section 3 of the paper. The approach is then applied to 50 bank branches. Conclusions appear in Section 7.

2. PPS and QPPS

In DEA, the efficiency for DMU_o, which is the analyzed object, is evaluated relatively by the other DMUs. We will deal with n DMUs with the input and output matrices $X = (x_{ij}) \in R^{m\times n}$ and $Y = (y_{rj}) \in R^{s\times n}$, respectively. We assume that the data set is non-negative, i.e. $X \geqslant 0$ and $Y \geqslant 0$. The production possibility set is defined as

$$T = \{(x, y) : x \geqslant X\lambda,\ y \leqslant Y\lambda,\ \lambda \geqslant 0\} \tag{1}$$

in which λ is a non-negative vector in R^n. Eq. (1) means that the more input values, smaller output values or both than those of given data can be productive. An input-oriented DEA model which exhibits constant returns to scale can be written as

$$\begin{aligned} \theta_o^* = \text{Min} \quad & \theta_o \\ \text{subject to:} \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leqslant \theta_o x_{io}, \quad i = 1, \ldots, m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geqslant y_{ro}, \quad r = 1, \ldots, s, \\ & \lambda_j \geqslant 0, \quad j = 1, \ldots, n, \end{aligned} \tag{2}$$

where x_{io} and y_{ro} are respectively the ith input and rth output for DMU_o. The set of DMUs can be partitioned into two groups: efficient DMUs and in-efficient DMUs. In order to discriminant the performance of efficient DMUs, we use the following super-efficiency DEA model, where DMU_o is excluded from the reference set [1]

$$\begin{aligned} \pi_o^* = \text{Min} \quad & \pi_o^{\text{super}} \\ \text{subject to:} \quad & \sum_{j=1, j\neq o}^{n} \lambda_j x_{ij} \leqslant \pi_o^{\text{super}} x_{io}, \quad i = 1, \ldots, m, \\ & \sum_{j=1, j\neq o}^{n} \lambda_j y_{rj} \geqslant y_{ro}, \quad r = 1, \ldots, s, \\ & \lambda_j \geqslant 0, \quad j = 1, \ldots, n,\ j \neq o. \end{aligned} \tag{3}$$

The quasi-production possibility set is defined as

$$P = \{(x, y) : x \leqslant X\lambda,\ y \geqslant Y\lambda,\ \lambda \geqslant 0\}. \tag{4}$$

T and P are closed and convex sets. It is to be noted that (4) is just opposite to (1) with respect to inequalities. The boundary points of T is referred to as efficient frontier, and we call the boundary points of P as anti-efficient frontier. To illustrate the sets T and P we use the two-dimensional input and one-dimensional output data. The data set is exhibited in Table 1.

Table 1
Data set for the numerical example

	DMU_j							
	1	2	3	4	5	6	7	8
Input 1	1	6	2	4	7	3	6	4
Input 2	6	1	4	2	3	7	6	4
Output	1	1	1	1	1	1	1	1