

Blow up solutions for one class of system of Pekar–Choquard type nonlinear Schrödinger equation [☆]

Jianqing Chen ^{a,b,*}, Boling Guo ^b

^a Department of Mathematics, Fujian Normal University, Fuzhou 350007, PR China

^b Institute of Applied Physics and Computational Mathematics, P.O. Box 8009, Beijing 100088, PR China

Abstract

We prove that firstly the existence of stationary solutions of the Pekar–Choquard system with the form

$$i\vec{\varphi}_t + \Delta \vec{\varphi} + K(\vec{\varphi}) + |\vec{\varphi}|^{p-2} \vec{\varphi} = 0, \quad (\text{PCS})$$

secondly the solutions of Cauchy problem of (PCS) with initial data close to the stationary solution (in a suitable sense) must blow up at finite time; finally the standing wave relating to the stationary solution of (PCS) is strongly unstable in the sense of Definition 4.2.

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1. Introduction

In this paper, we consider the following system of Pekar–Choquard type nonlinear Schrödinger equation with density-functional correct term in 3 dimensions:

$$i\vec{\varphi}_t + \Delta \vec{\varphi} + K(\vec{\varphi}) + |\vec{\varphi}|^{p-2} \vec{\varphi} = 0, \quad x \in \mathbb{R}^3, \quad t \in \mathbb{R}_+, \quad (1.1)$$

$$\vec{\varphi}(x, 0) = \vec{\varphi}_0(x), \quad x \in \mathbb{R}^3, \quad (1.2)$$

where $i = \sqrt{-1}$, $p > 2$ and Δ is the Laplace operator. $\vec{\varphi}(x, t) = (\varphi_1(x, t), \dots, \varphi_N(x, t))$ is a N component unknown complex functional vector,

$$K(\vec{\varphi}) = \int V(x - y) \Phi(x, y, t) \overline{\vec{\varphi}(y, t)} dy \quad (1.3)$$

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* Corresponding author. Address: Department of Mathematics, Fujian Normal University, Fuzhou 350007, PR China.

E-mail address: jqchen@fjnu.edu.cn (J. Chen).

and $\Phi(x, y, t) = (\Phi_{jk}(x, y, t))$ is a $N \times N$ complex valued functional matrix with

$$\Phi_{jk}(x, y, t) = \varphi_j(x, t)\varphi_k(y, t) - \varphi_k(x, t)\varphi_j(y, t), \quad j, k = 1, 2, \dots, N.$$

By $\overline{\varphi(y, t)}$ we denote the complex conjugate of $\varphi(y, t)$. $V(x)$ is a suitable integral kernel defined in $\mathbb{R}^3$. System (1.1) can also be written as following componentwise:

$$i\varphi_{m,t} + \Delta\varphi_m + K_m(\vec{\varphi}) + |\vec{\varphi}|^{p-2}\varphi_m = 0, \quad m = 1, 2, \dots, N, \quad (1.4)$$

where

$$\begin{aligned} K_m(\vec{\varphi}) &= \int \sum_{k=1}^N (\varphi_m(x, t)\varphi_k(y, t) - \varphi_k(x, t)\varphi_m(y, t))V(x-y)\overline{\varphi_k(y, t)} \, dy \\ &= \int \varphi_m(x, t)|\vec{\varphi}(y, t)|^2 V(x-y) \, dy - \int \sum_{k=1}^N \varphi_k(x, t)\varphi_m(y, t)\overline{\varphi_k(y, t)}V(x-y) \, dy. \end{aligned}$$

Equations of this kind appears in various physical areas. For example, the nonlinear Schrödinger equation with nonlocal interaction

$$i\varphi_t + \Delta\varphi + \varphi \int V(x-y)|\varphi(y, t)|^2 \, dy = 0 \quad (x, t) \in \mathbb{R}^3 \times \mathbb{R}_+ \quad (\text{SE})$$

has been proposed by Efinger [4]. (SE) can be looked upon as the classical limit, in the sense of Hartree type approximations, of the field equation which describes a quantum mechanical nonrelativistic particle system interacting through a nonlocal potential. Further example was provided in a Hartree–Fock theory for a one component plasma and (SE) has been studied extensively, see e.g. [2,8]. Hartree–Fock equation with N ($N \geq 2$) body interaction was obtained by Dirac and Hartree–Fock–Choquard system of nonlinear Schrödinger equations with density-functional correct term was proposed by Deb et al. [3,6].

The main goals here are as follows:

In Section 2, we prove the existence of stationary solutions, i.e., solutions of the form $\varphi_m(x, t) = e^{i\lambda_m t}u_m(x)$, of (1.4). Indeed, we use variational methods to prove that (1.1) possesses a solution $\vec{\varphi}(x, t) = (e^{i\lambda_1 t}u_1(x), \dots, e^{i\lambda_N t}u_N(x))$ with $\vec{u} = (u_1, \dots, u_N)$ minimizing the Euler–Lagrange functional over a set of Nehari type, see Theorem 2.4. In Section 3, we firstly look upon the local solution of (1.1) and (1.2) as a flow and construct a set $\mathcal{S}$ which is invariant under this flow. Secondly we prove that if the initial data is contained in $\mathcal{S}$ and close (in a suitable sense) to $\vec{u}$, then the solution of (1.1) and (1.2) must blow up at finite time, see Theorem 3.4. In Section 4, we prove that the standing wave $(e^{i\lambda_1 t}u_1(x), \dots, e^{i\lambda_N t}u_N(x))$ is strongly unstable in the sense of Definition 4.2, see Theorem 4.3.

We end this introduction by some assumptions and notations. Throughout this paper, we assume from the physical point of view that the integral kernel

$$V(x) = 1/|x|^\alpha, \quad 2 < \alpha < 3 \quad \text{and} \quad 4 \leq p < 6.$$

All integrals are taken over $\mathbb{R}^3$ unless stated otherwise. dx will be omitted if no confusion occurs. $(\cdot, \cdot)_{L^2}$ ($\|\cdot\|_{L^2}$) denotes $L^2(\mathbb{R}^3)$ or $(L^2(\mathbb{R}^3))^N$ inner product (norm) which will be understood from the context. $\rightarrow$ denotes the strong convergence and $\rightharpoonup$ the weak convergence. Re denotes the real part and Im the imaginary part.

2. Stationary solutions

In this section, we use variational methods to prove the existence of stationary solutions of (1.1). More precisely, considering a solution of (1.1) of the form $\vec{\varphi}(x, t) = (e^{i\lambda_1 t}u_1(x), \dots, e^{i\lambda_N t}u_N(x))$ with $\lambda_m > 0$ and u_m real functions, then (1.4) becomes

$$-\Delta u_m + \lambda_m u_m = K_m(\vec{u}) + |\vec{u}|^{p-2}u_m, \quad \vec{u} = (u_1, \dots, u_N), \quad (2.1)$$

$$K_m(\vec{u}) = \int \left[u_m(x)|\vec{u}(y)|^2 - \sum_{k=1}^N u_k(x)u_m(y)u_k(y) \right] V(x-y) \, dy. \quad (2.2)$$

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