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Permanence and stability of equilibrium for a two-prey one-predator discrete model ☆

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Abstract

In this letter, a predator-prey system of two-prey one-predator discrete model is investigated. It is proved that the system is permanence under some appropriate conditions. By Jacobian matrix method, a sufficient and necessary condition is derived for the local asymptotic stability of a equilibrium of the system. Meanwhile, we give a suitable example for supporting our theoretical result.

Keywords: Predator-prey system; Discrete model; Stability; Permanence

1. Introduction

In recent years the dynamic relationship between predators and their prey has long been and will continue to be one of the dominant themes in both ecology and mathematical ecology. In the theoretical ecology, permanence and stability of equilibrium of the predator-prey model are very important. There are extensive literature related to these topics for differential equation models (see [1,2,5–9] and the references cited therein). Most recently, Sikder [1] considered uniform persistence of the following two prey-one predator continuous time model

$$\begin{cases} \frac{dN_1}{dt} = r_1N_1\left(1-\frac{N_1}{k_1}\right) - \frac{a_1N_1N_3}{1+a_1h_1N_1+\beta a_2h_2N_2}, \\ \frac{dN_2}{dt} = r_2N_2\left(1-\frac{N_2}{k_2}\right) - \frac{\beta a_2N_2N_3}{1+a_1h_1N_1+\beta a_2h_2N_2}, \\ \frac{dN_3}{dt} = N_3\left(\frac{c(a_1e_1N_1+\beta a_2e_2N_2)}{1+a_1h_1N_1+\beta a_2h_2N_2} - d\right), \end{cases} \tag{1.1}$$

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where N_1, N_2 and N_3 are the dnsities of prey species 1, prey species 2 and predator, respectively. For prey species i, r_i is the maximum per capita growth rate, k_i is the carrying capacity, a_i is the effective search rate per unit time, e_i is its net energy content, h_i is the handling time. The parameter β denotes the probability that a predator attacks the prey type 2. The parameter c is a proportionality constant converting energy intake per predator to per capita reproduction of predators and d is the predator mortality rate.

Naturally, the discrete time models are more appropriate than the continuous ones when the size of the population is rarely small or the population has non-overlapping generations. Recently, there has been a tendency for some researchers in the field of difference equations to develop some new methods which are analogous to those used in the study of differential equations (see, e.g., [3,4,10–20] and the references therein).

The main purpose of this paper is to study the following general discrete analogue of continuous time model (1.1)

$$\begin{cases} N_1(n+1) = N_1(n)\exp\left[r_1(n) - \frac{r_1(n)N_1(n)}{k_1(n)} - \frac{a_1(n)N_3(n)}{1+a_1(n)h_1(n)N_1(n)+\beta a_2(n)h_2(n)N_2(n)}\right], \\ N_2(n+1) = N_2(n)\exp\left[r_2(n) - \frac{r_2(n)N_2(n)}{k_2(n)} - \frac{\beta a_2(n)N_3(n)}{1+a_1(n)h_1(n)N_1(n)+\beta a_2(n)h_2(n)N_2(n)}\right], \\ N_3(n+1) = N_3(n)\exp\left[-d(n) + \frac{c(a_1(n)e_1(n)N_1(n)+\beta a_2(n)e_2(n)N_2(n))}{1+a_1(n)h_1(n)N_1(n)+\beta a_2(n)h_2(n)N_2(n)}\right]. \end{cases} \tag{1.2}$$

For simplitity, we nondimensionalize the system (1.2) with the following scaling:

$$x_1 \to \frac{N_1}{k_1}, \quad x_2 \to \frac{N_2}{k_2}, \quad y \to N_3,$$

and then obtain the form

$$\begin{cases} x_1(n+1) = x_1(n)\exp\left[r_1(n) - r_1(n)x_1(n) - \frac{a_1(n)y(n)}{1+A(n)x_1(n)+B(n)x_2(n)}\right], \\ x_2(n+1) = x_2(n)\exp\left[r_2(n) - r_2(n)x_2(n) - \frac{\beta a_2(n)y(n)}{1+A(n)x_1(n)+B(n)x_2(n)}\right], \\ y(n+1) = y(n)\exp\left[-d(n) + \frac{C(n)x_1(n)+D(n)x_2(n)}{1+A(n)x_1(n)+B(n)x_2(n)}\right], \end{cases} \tag{1.3}$$

where

$$A(n) = a_1(n)h_1(n)k_1(n), \quad B(n) = \beta a_2(n)h_2(n)k_2(n),$$
$$C(n) = ca_1(n)e_1(n)k_1(n), \quad D(n) = c\beta a_2(n)e_2(n)k_2(n).$$

Throughout this paper, we will assume that $r_1(n)$, $r_2(n)$, $a_1(n)$, $a_2(n)$, $A(n)$, $B(n)$, $C(n)$, $D(n)$ and $d(n)$ are bounded nonnegative sequences, and use the following notations for any bounded sequence $u(n)$:

$$\bar{u} = \sup_{n\in\mathbb{N}} u(n), \quad \underline{u} = \inf_{n\in\mathbb{N}} u(n),$$

where $\mathbb{N}$ is the set of nonnegative integer numbers. By the biological meaning, we will focus our discussion on the positive solutions of (1.3). Thus, we require that $x_i(0) > 0$ $(i = 1,2)$, $y(0) > 0$.

In this letter, we will study the permanence and the local stability of equilibrium for this discrete model. Finally, a suitable example is given to illustrate the feasibility of the conditions of our theorem.

2. Permanence

In this section, we will establish a permanence result for system (1.3). Firstly, we introduce a definition and state some lemmas which will be useful to establish our main results.

Definition 2.1. System (1.3) is said to be permanence if there exist two positive vectors m and M such that

$$m \leqslant \liminf_{n\to\infty}(x_1(n),x_2(n),y(n)) \leqslant \limsup_{n\to\infty}(x_1(n),x_2(n),y(n)) \leqslant M,$$

for any solution $x(n) = (x_1(n),x_2(n),y(n))$ of (1.3).