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New solitary wave solutions to the modified forms of Degasperis–Procesi and Camassa–Holm equations

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Abstract

In this work, we establish new solitary wave solutions to the modified forms of Degasperis–Procesi and Camassa–Holm equations. Unlike the standard Degasperis–Procesi and Camassa–Holm equations, where multi-peakon solutions arise, the modified forms cause a change in the characteristic of these solutions and change it to bell-shaped solitons, periodic, and complex solutions. The extended tanh method, the rational hyperbolic functions method, and the rational exponential functions method are employed to reveal these new solutions.
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1. Introduction

The Camassa–Holm (CH) equation

$$u_t - u_{xxt} + 3uu_x = 2u_x u_{xx} + uu_{xxx} \tag{1}$$

has been investigated in the literature in [1–11] and the references therein. However, changing the coefficients 3 and 2 in (1) to 4 and 3, respectively, gives the Degasperis–Procesi (DP) equation

$$u_t - u_{xxt} + 4uu_x = 3u_x u_{xx} + uu_{xxx}. \tag{2}$$

The CH and the DP equations (1) and (2) are bi-Hamiltonian and have an associated isospectral problem [1]. Both equations are formally integrable [1] by means of the scattering/inverse scattering approach. Moreover, both equations admit peaked solitary wave solutions. Eqs. (1) and (2) present similarities although they are truly different. The isospectral problem for DP equation is of third order, whereas the CH equation admits a second order isospectral problem [1]. The CH equation (1) is a shallow water equation and was originally derived as an approximation to the incompressible Euler equation and found to be completely integrable with a Lax pair [2]. The DP equation (2) can be considered as a model for shallow-water dynamics and found to be completely integrable as stated before.

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Many powerful methods, such as inverse scattering method, Bäcklund transformation, the Wadati trace method, Hirota bilinear forms, pseudo spectral method, the tanh–sech method [12–16], the sine–cosine method [17–25], Jacobi elliptic functions [26,27], and the Riccati equation expansion method were used to investigate these types of equations. Recently, Shen et al. [4] studied the global behavior of the DP equation by using bifurcation theory of dynamical system.

The CH equation has peaked solitary wave solutions of the form

$$u(x,t) = c\mathrm{e}^{(-|x-ct|)}, \tag{3}$$

where c is the wave speed. The name "peakon", that is, solitary wave with slope discontinuities, was used to single them from general solitary wave solutions since they have a corner at the peak of height c.

It is the aim of this work to further complement the studies of the DP and the CH equations. More precisely, we will investigate modified forms of the DP and the CH equations given by

$$u_t - u_{xxt} + 4u^2u_x = 3u_xu_{xx} + uu_{xxx} \tag{4}$$

and

$$u_t - u_{xxt} + 3u^2u_x = 2u_xu_{xx} + uu_{xxx}, \tag{5}$$

respectively. It is clear that the nonlinear convection term uu_x in (2) and (1) has been changed to u^2u_x in (4) and (5). We aim in this work to use these modified forms as a vehicle to explore the change in the physical structure of the solution from peakons solutions (3) to bell-shaped solitary wave solutions and to solitons structures in the form of a ration of exponential functions. To achieve our goal, the extended tanh method, the rational hyperbolic functions methods, and the rational exponential functions method will be used to conduct our analysis. In what follows, we highlight the main steps of these methods.

2. The methods

A PDE

$$P(u, u_t, u_x, u_{xx}, u_{xxx}, \ldots) = 0 \tag{6}$$

can be converted to an ODE

$$Q(u, u', u'', u''', \ldots) = 0 \tag{7}$$

upon using a wave variable $\xi = (x - ct)$. Eq. (7) is then integrated as long as all terms contain derivatives where integration constants are considered zeros.

2.1. The extended tanh method

The tanh method developed by Malfliet in [12], and used in [13–16] among many others, introduces an independent variable

$$Y = \tanh(\mu\xi), \quad \xi = x - ct \tag{8}$$

is introduced that leads to the change of derivatives:

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}\xi} &= \mu(1 - Y^2)\frac{\mathrm{d}}{\mathrm{d}Y}, \\ \frac{\mathrm{d}^2}{\mathrm{d}\xi^2} &= -2\mu^2 Y(1 - Y^2)\frac{\mathrm{d}}{\mathrm{d}Y} + \mu^2(1 - Y^2)^2\frac{\mathrm{d}^2}{\mathrm{d}Y^2}. \end{aligned} \tag{9}$$

The extended tanh method admits the use of the finite expansion

$$u(\mu\xi) = S(Y) = \sum_{k=0}^{M} a_k Y^k + \sum_{k=1}^{M} b_k Y^{-k}, \tag{10}$$

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