Available online at www.sciencedirect.com
ScienceDirect
ELSEVIER
Applied Mathematics and Computation 186 (2007) 159–174
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

Remarks on the Kalman filtering simulation and verification

Dah-Jing Jwo

Department of Communications and Guidance Engineering, National Taiwan Ocean University, 2 Peining Rd., Keelung 202, Taiwan, ROC

Abstract

The purpose of this paper is to convey the readers several useful remarks for understanding, simulating and verifying the Kalman filter (KF) computer codes. A tutorial, example-based approach is employed to present several KF issues of considerable importance in engineering practice, and to suggest some check points on Kalman filtering verification process. Some illustrative examples are accompanied where necessary to the readers for better understanding the fundamental basis and for enhancing the reliability (correctness) of the self-developed computer codes before larger, complicated KF designs are performed. Notes on two forms of discrete-time Kalman filter loop are pointed out. Methods for determining the process noise covariance matrix are provided. Simulation of the dynamic process is discussed. Guidelines for verification of filtering solutions are provided, which cover (1) the consistency check between the discrete-time to the continuous-time covariance and gain matrices; (2) evaluation of estimator optimality with sensitivity analysis and consistency check between theoretical and simulation results.

Keywords: Kalman filter; Simulation; Verification; Optimality evaluation; Sensitivity analysis

1. Introduction

The Kalman filter [1–6] (KF) or its nonlinear version, extended Kalman filter (EKF), has been the most well known sequential data assimilation scheme for solving the Wiener problem in a generally easier way. It has been applied in the areas as diverse as aerospace, marine navigation, radar target tracking, control systems, manufacturing, and many others. For the aerospace navigation applications, it has been very popular in GPS/INS and GPS stand-alone navigation designs and is recognised as the navigation's integration workhorse. A navigation filter is commonly designed by use of a Kalman filter to estimate the vehicle state variables and suppress the navigation measurement noise. The Kalman filter not only works well in practice, but also it is theoretically attractive because it has been shown that it is the filter that minimizes the variance of the estimation mean square error.

Studying the operation of the Kalman filter leads to an appreciation of the inter-disciplinary nature of system engineering. However, implementation of the Kalman filter is a challenge to some system designers. A

E-mail address: djjwo@mail.ntou.edu.tw

0096-3003/$ - see front matter
doi:10.1016/j.amc.2006.06.124

deeper understanding of the theory and awareness of practical implementation can only be experienced by employing the filter in practical situation. It may not be such a difficult task to develop a KF computer code. However, it is a challenge to assure its reliability. Even after constructing the program code, some designers may not be able to ensure the correctness of the computer code developed by them. Based on the consideration, several practical remarks are pointed out in this article for clarifying some fundamental concept and conveying some important phenomena. For better illustration, numerical examples are provided where necessary to the readers for better understanding the fundamental basis. The remarks presented in this paper are beneficial to the KF designers, which can be employed as guidelines for developing reliable Kalman filter computer codes.

This paper is organized as follows. In Section 2, additional notes on discrete-time Kalman filter (DTKF) loop are pointed out. In Section 3, determination of the process noise covariance matrix is presented. Simulation of the dynamic process is discussed in Section 4. In Section 5, relation of the DTKF to the continuous Kalman filter (CKF) is shown. The evaluation of estimator optimality for verification of minimum variance optimality with sensitivity analysis and consistency check is provided in Section 6. The conclusion is given in Section 7.

2. Additional notes on discrete-time Kalman filter loop

Consider a dynamical system whose state is described by a linear, vector differential equation. The process model and measurement model are represented as

$$\text{Process model}: \dot{\mathbf{x}} = \mathbf{F}\mathbf{x} + \mathbf{G}\mathbf{u}, \tag{1}$$

$$\text{Measurement model}: \mathbf{z} = \mathbf{H}\mathbf{x} + \mathbf{v}, \tag{2}$$

where the vectors $\mathbf{u}(t)$ and $\mathbf{v}(t)$ are both white noise sequences with zero means and mutually independent:

$$E[\mathbf{u}(t)\mathbf{u}^{\mathrm{T}}(\tau)] = \mathbf{Q}\delta(t-\tau), \tag{3a}$$

$$E[\mathbf{G}(t)\mathbf{u}(t)(\mathbf{G}(t)\mathbf{u}(\tau))^{\mathrm{T}}] = \mathbf{G}\mathbf{Q}\mathbf{G}^{\mathrm{T}}\delta(t-\tau), \tag{3b}$$

$$E[\mathbf{v}(t)\mathbf{v}^{\mathrm{T}}(\tau)] = \mathbf{R}\delta(t-\tau), \tag{4}$$

$$E[\mathbf{u}(t)\mathbf{v}^{\mathrm{T}}(\tau)] = \mathbf{0}, \tag{5}$$

where $\delta(t-\tau)$ is the Dirac delta function, $E[\cdot]$ represents expectation, and superscript "T" denotes matrix transpose.

2.1. The continuous Kalman filter

The state estimate equation of the continuous Kalman filter equations is represented as

$$\dot{\hat{\mathbf{x}}} = \mathbf{F}\mathbf{x} + \mathbf{K}(\mathbf{z} - \mathbf{H}\hat{\mathbf{x}}). \tag{6}$$

The propagation of the error for a continuous Kalman filter can be described by the Riccati equation,

$$\dot{\mathbf{P}} = \mathbf{F}\mathbf{P} + \mathbf{P}\mathbf{F}^{\mathrm{T}} - \mathbf{P}\mathbf{H}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{H}\mathbf{P} + \mathbf{G}\mathbf{Q}\mathbf{G}^{\mathrm{T}} \tag{7}$$

and the continuous filter gain is obtained through the calculation

$$\mathbf{K} = \mathbf{P}\mathbf{H}^{\mathrm{T}}\mathbf{R}^{-1}. \tag{8}$$

The discrete filter gain and continuous filter gain are related by

$$\mathbf{K} = \frac{\mathbf{K}_k}{\Delta t}, \tag{9}$$

where $\Delta t = t_{k+1} - t_k$ represents the sampling period. When the system reaches steady-state, $\dot{\mathbf{P}} = \mathbf{0}$, Eq. (7) becomes an Algebraic Riccati Equation (ARE), which can be solved for the steady-state minimum covariance matrix.