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Numerical solution for biharmonic equation using multilevel radial basis functions and domain decomposition methods

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Abstract

Biharmonic equation has significant applications in physics and engineering, but is difficult to solve due to the existing fourth order derivatives. One of the domain-type mashless methods is obtained by simply applying the radial basis functions (RBFs) as a direct collocation, which has shown to be effective in solving complicated physical problems with irregular domains. In this paper, we utilize overlapping domain decomposition and multilevel RBF methods for solving biharmonic equation. Numerical results indicate that these two methods circumvent the ill-conditioning problem resulted from using the radial basis function as a global interpolant.

Keywords: Radial basis functions; Meshless method; Multilevel RBF; Overlapping domain decomposition; Biharmonic equation

1. Introduction

Consider the two-dimensional biharmonic equation

$$\nabla^4 u(x,y) = \frac{\partial^4 u}{\partial x^4} + \frac{\partial^4 u}{\partial y^4} + 2\frac{\partial^4 u}{\partial x^2 \partial y^2} = f(x,y), \quad (x,y) \in \Omega \tag{1.1}$$

and Dirichlet boundary conditions

$$u = f_1(x,y), \quad \frac{\partial u}{\partial n} = f_2(x,y), \qquad (x,y) \in \partial\Omega.$$

Here Ω is a two-dimensional simply-connected domain with a piecewise smooth boundary $\partial\Omega$, and $u_n = \frac{\partial u}{\partial n}$ is the outward normal derivative of u on $\partial\Omega$.

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Table 1
Globally supported radial basis functions

$\varphi(r) = \sqrt{r^2 + c^2}$	Multiquadric
$\varphi(r) = (r^2 + c^2)^{-1/2}$	Inverse multiquadric
$\varphi(r) = r^2 \ln(r)$	Thin plate spline
$\varphi(r) = e^{-c^2 r^2}$	Gaussian

Various approaches for the numerical solution of the biharmonic equation have been considered in the literature. For example, numerical solutions of this equation is usually obtained through the use of the finite difference method (FDM) because of the ease of grid generation and the dissipative characteristics of the method, which results in a more stable solution. However, the FDM usually involves a rectangular grid system, which makes it very difficult to model the detailed topographic features of an irregular domain. Although the finite element method (FEM) can accommodate a more flexible gridwork and has been used as an alternative solution scheme for this equation, the finite element solution is not as stable as the finite difference solution and usually requires the use of nonphysical dissipation. Furthermore, the generation of a finite element grid with several thousand nodes and with elements of various sizes, shapes and orientations is not a trivial task.

In the last decade, the development in applying the radial basis functions (RBFs) as a truly meshless method for approximating the solutions of PDEs has drawn the attention of many researchers in science and engineering [9,10]. Table 1 lists some of the globally supported RBFs. As usual, we use $r = \|\cdot\|$ (the Euclidean norm), and c is a parameter to be set by the user. In 1971, the multiquadric (MQ) was first developed by Hardy [13] as a multidimensional scattered interpolation method in modeling the earth's gravitational field. It was not recognized by most of the academic researchers until Franke [8] published a review paper in the evaluation of 29 2D interpolation methods whereas MQ was ranked the best based on its accuracy, visual aspect, sensitivity to parameters, execution time, storage requirements, and ease of implementation. One of the domain-type meshless methods, the so-called Kansa's method developed by Kansa in 1990 [15], is obtained by directly collocating the RBFs, particularly the multiquadric (MQ), for the numerical approximation of the solution. Kansa's method was recently extended to solve various ordinary and partial differential equations including nonlinear Burgers' equation with shock wave, shallow water equations for tide and current simulation, heat transfer problems, and free boundary problems. Fasshauer [3] later modified Kansa's method to a Hermite-type collocation method for the solvability of the resultant collocation matrix.

In this paper, we solve the biharmonic equation by collocation with radial basis functions. The paper is organized as follows. In Sections 2 and 3, a brief introduction to both Kansa's and Hermite approaches is given. In Sections 4–6 we outline the overlapping domain decomposition and multilevel radial basis functions methods. Numerical results are given in Section 7.

2. Kansa's approach

In this context we are given data (x_j, f_j), $j = 1, \ldots, N$, $x_j \in \mathbb{R}^d$, where we can think of the values f_j being sampled from a function $f : \mathbb{R}^d \to \mathbb{R}$. The goal is to find an interpolant of the form

$$s(x) = \sum_{k=1}^{N} c_k \varphi(\|x - x_k\|), \quad x \in \mathbb{R}^d \tag{2.1}$$

such that

$$s(x_j) = f_j, \quad j = 1, \ldots, N.$$

The solution of this problem leads to a linear system $Ac = f$ with the entries of A given by

$$A_{j,k} = \varphi(\|x_j - x_k\|), \quad j, k = 1, \ldots, N. \tag{2.2}$$

Clearly, there exists a unique solution if and only if A is non-singular. The challenge now is to find the largest possible class of functions φ for which this is true. This is still an unsolved problem. However, e.g., in [2,16,17]