Available online at www.sciencedirect.com
ScienceDirect
Applied Mathematics and Computation 186 (2007) 460–468
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

# Dynamics for a class of general hematopoiesis model with periodic coefficients

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**Abstract**

Sufficient conditions are obtained for the existence and global attractivity of a unique positive periodic solution $\tilde{x}(t)$ of

$$x'(t) = -a(t)x(t) + \frac{b(t)}{1 + x^n(t - \tau(t))}, \quad t > 0, \tag{$*$}$$

where $n > 1$, $a$ and $b$ are continuous positive periodic function. Also, some sufficient conditions are established for oscillation of all positive solutions of ($*$) about $\tilde{x}(t)$. For the proof of existence and uniqueness of $\tilde{x}(t)$, the method used here is better than contraction mapping principle.


*Keywords:* Positive periodic solution; Cone; Oscillation; Global attractivity

## 1. Introduction

The purpose of the present paper is to investigate the existence and global attractivity of unique positive solution $\tilde{x}(t)$ of the following equation with periodic coefficients:

$$x'(t) = -a(t)x(t) + \frac{b(t)}{1 + x^n(t - \tau(t))}, \quad n > 1. \tag{1.1}$$

Meanwhile, the oscillation of every positive solution of (1.1) about $\tilde{x}(t)$ will be also studied. It is necessary to consider behaviors of all positive solutions of (1.1). In fact, (1.1) is one of generations of hematopoiesis models

$$x'(t) = -ax(t) + \frac{b}{1 + x^n(t - \tau)}, \quad n > 0, \tag{1.2}$$

which (after some transformations) was first proposed by Mackey and Glass [10] to describe some physiological control systems. The global attractivity of positive steady solution $K$ of (1.2) and the oscillatory behavior

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0096-3003/$ - see front matter 
doi:10.1016/j.amc.2006.07.109

of all positive solution about $K$ have been studied. See Karakostas et al. [5], Kuang [7], Saker [11] and Zaghrout et al. [16], for instance. For further investigation in this area, for example, the delay differential equations

$$x'(t) = -a(t)x(t) + \frac{b(t)x^m(t-k\omega)}{1+x^n(t-k\omega)}, \tag{1.3}$$

where $m = 0,1$ and $n > m$, $a(t)$ and $b(t)$ are positive $\omega$-periodic functions, and

$$x'(t) = -a(t)x(t) + b(t)\int_0^\infty K(s)\frac{1}{1+x^n(t-s)}\mathrm{d}s, \quad n > 0, \tag{1.4}$$

where $K$: $[0,\infty) \to [0,\infty)$, and $a(t)$ and $b(t)$ are positive $\omega$-periodic functions. In [12], the author not only obtained that (1.3) had a unique positive periodic solution $\tilde{x}(t)$ under some assumptions when $k = 0$ but also studied oscillation of all positive solutions of (1.3) about $\tilde{x}(t)$ and global attractivity of $\tilde{x}(t)$. And some sufficient conditions have been obtained by Yang and Weng [15] for the existence and global attractivity of a positive periodic solution of (1.4).

As far as we know, though the existence of positive periodic solution of (1.1) has been already done by Jiang and Wei [4] and Wan et al. [13], other behaviors of solutions of (1.1) have never been studied. Motivated by [2,6,9], in this paper we shall investigate the existence and uniqueness of positive periodic solution $\tilde{x}(t)$ of (1.1) by using a fixed theorem in cone which is different from that used in [4,13,14] and also show that the method used here is better than contraction mapping principle. And we shall prove that if $\tau(t) \equiv \tau$, then $\tilde{x}(t)$ is a global attractor and give some sufficient conditions to guarantee that every positive solution oscillates about $\tilde{x}(t)$.

Note that if $\tau(t) = k\omega$ in (1.1), then (1.1) reduces to (1.3) for $m = 0$ and $n > 1$. For this case, our main results complement that in [12].

Throughout this paper, in (1.1), we always suppose that $a(t)$, $b(t)$ and $\tau(t)$ are positive continuous $\omega$-periodic functions on $R$.

For convenience, we also need to introduce a few notations. Let

$$G(t,s) = \frac{\exp(\int_t^s a(r)\mathrm{d}r)}{\exp(\int_0^\omega a(r)\mathrm{d}r) - 1}, \quad s \in [t, t+\omega],$$

$$N = G(t,t) = \min_{t\in[0,\omega],s\in[t,t+\omega]}\{G(t,s)\} \leqslant \max_{t\in[0,\omega],s\in[t,t+\omega]}\{G(t,s)\} = G(t,t+\omega) = M,$$

$$h^* = \max_{t\in[0,\omega]} h(t), \quad h_* = \min_{t\in[0,\omega]} h(t), \quad \text{and} \quad h = \int_0^\omega h(s)\mathrm{d}s,$$

where $h(t)$ is a continuous $\omega$-periodic function on $R$.

In view of the actual applications of (1.1), we shall only consider the solutions of (1.1) with initial condition

$$x(s) = \varphi(s) \quad \text{for } s \in [-\tau^*, 0], \quad \varphi \in C([-\tau^*, 0], [0,\infty)), \quad \varphi(0) > 0. \tag{1.5}$$

## 2. Some definitions and lemmas

The proofs of the main results in our paper are based on an application of fixed point theorem in cone (see [3]). To make use of fixed point theorem in cone, firstly, we need to introduce some definitions and lemmas. Let $X$ be a real Banach space, $P$ is a cone of $X$. The semi-order induced by the cone $P$ is denoted by "$\leqslant$". That is, $x \leqslant y$ if and only if $y - x \in P$ for any $x, y \in P$.

**Definition 2.1.** A cone $P$ of $X$ is said to be normal if there exists a positive constant $\delta$ such that $\|x + y\| \geqslant \delta$ for any $x, y \in P$, $\|x\| = \|y\| = 1$.

**Definition 2.2.** $P$ is a cone of $X$ and $A$: $P \to P$ is an operator. $A$ is called decreasing, if $\theta \leqslant x \leqslant y$ implies $Ax \geqslant Ay$, where $\theta$ denotes the zero element of $X$.