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The boundary penalty method for the diffusion equation subject to the specification of mass ☆

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Abstract

In this paper, we concern with a new boundary penalty finite element method for the one-dimensional time-dependent diffusion equation subject to the specification of mass. Instead of solving the diffusion equation directly, we will use the finite element method to solve a boundary penalty problem which is derived from the diffusion equation and analyse the error estimate of the semi-discrete and fully discrete approximations when the penalty parameter ε is sufficiently small. The numerical experiments show that our method is very effective and supports our analysis.
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Keywords: Penalty method; Diffusion equation; Specification of mass; Finite element method; Error estimate

1. Introduction

In this paper, we consider the numerical solution of the diffusion equation

$$u_t = u_{xx}, \quad 0 < x < 1, \ 0 < t \leqslant T \tag{1}$$

subject to the initial condition

$$u(x, 0) = f(x), \quad 0 < x < 1 \tag{2}$$

and the boundary condition

$$u_x(1, t) = g(t), \quad 0 < t \leqslant T \tag{3}$$

and the non-local boundary condition

$$E(t) = \int_0^b u(x, t)\,\mathrm{d}x, \quad 0 < b < 1, \ 0 < t \leqslant T, \tag{4}$$

where $f(x)$, $g(t)$ and $E(t)$ are known functions, T and b are given positive constant.

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Many physical phenomena were formulated into non-local mathematical models (1)–(4) [3–5]. For example, when u denotes the concentration of a chemical which is diffusing in a straight glass tube with x measured in the direction of the axis of the tube, the electric signal produced by a light beam passing through the tube at the right angles to the tube between $x = 0$ and $x = b$ is proportional to $E(t)$, which is the total mass of the chemical in $[0, b]$ at time t. Likewise, when u denotes temperature in heat conduction problems, $E(t)$ can represents an internal energy content of the region $0 < x < b$ at time t. There are also many other fields, where the non-local problems are widely used, such as the transport of reactive and passive contaminates in aquifers, active interdisciplinary research of mathematicians, engineers, and life scientists and so on. The reader can find the derivation of non-local mathematical models and for the precise hypotheses and analysis in [6,7].

The existence, uniqueness and continuous dependence of the classical solution on the data for this linear diffusion equation subject to the specification of mass (1)–(4) and some other similar semi-linear, quasi-linear and non-linear diffusion equations with temperature boundary conditions and all the other type of boundary conditions were considered by many researchers. Cannon and van der Hoek [1] demonstrated that problems (1)–(4) possess a unique solution that depends continuously upon data according to the equivalence of the existence and unicity between the solution of problems (1)–(4) and the solution of a Volterra integral equation of the second kind. For other type of boundary conditions, Cannon [8] and Deckert and Maple [9] considered the problems similar to (1)–(4) with temperature boundary conditions, and demonstrated the existence, uniqueness and continuous dependence upon the data. Ionkin [10] considered a special case of (1)–(4).

The numerical solution of diffusion equations with non-local boundary specifications is currently an active area of research. There has been growing interest in developing numerical techniques and schemes for the numerical solution of diffusion equations with non-local boundary specifications [11]. Cannon and van der Hoek [1] considered a finite difference scheme for the initial value problem for the one-dimensional diffusion equation and a trapezoid rule approximation for the non-local boundary condition. Cannon et al. [12] and Cannon and van der Hoek [13] proposed the implicit finite difference schemes for the one- dimensional diffusion equation subject to the specification of mass. Dehghan [14,15] considered the fully implicit finite difference methods for two-dimensional and three-dimensional diffusion with a non-local boundary condition, and in [16] the application of the method of lines (MOL) to such problems is considered. The familiar finite element and boundary element methods have also been used by many authors, Cannon et al. [17] considered a Galerkin procedure for the diffusion equation subject to the specification of mass. Ewing et al. [18] used finite volume element approximations of the similar problems.

This paper focuses on the boundary penalty method for the one-dimensional time-dependent diffusion equation subject to the specification of mass (1)–(4). The penalty method are widely used for the elliptic equations, such as the stationary stokes equation [19,20], the unsteady Navier–Stokes equations [21] and the time-dependent Navier–Stokes equations [22]. Cheng and Wasim Shaikh [23] proposed an iterative penalty method for addressing the Stokes equations by using a "not very small" penalty parameter to avoid the unstable computation by iteration. In this paper, we will use the boundary penalty method for the parabolic equations. We will propose a boundary penalty problem whose solution converges to the solution of the one-dimensional time-dependent diffusion equation subject to the specification of mass (1)–(4) when the penalty parameter ε tends to zero, and use the finite element method to solve the corresponding boundary penalty problem with a small penalty parameter ε. The error estimates for the semi-discrete and fully discrete will be analysed. The numerical experiments will show the efficiency of our proposed method.

The remainder of the paper is organized as follows. In Section 2, we will propose a boundary penalty problem corresponding to the diffusion equation subject to the specification of mass (1)–(4). The convergence property of the solution when $\varepsilon \to 0$ will be stated. In Section 3, the corresponding boundary penalty problem will be solved by the finite element method with a small penalty parameter ε. We will discuss the error estimate for the semi-discrete and fully discrete schemes. In Section 4, some numerical examples will be given to show that our proposed boundary penalty method and the finite element approximation scheme are effective and powerful when we choose a sufficiently small penalty parameter $\varepsilon < \frac{h}{10}$, where h is the spatial step.

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