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Mean response time for a $G/G/1$ queueing system: Simulated computation

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Abstract

Mean response time, denoted by r, plays an important role of system characteristics in queueing models. In this paper we developed a data-based recursion relation to compute a sequence of response times, and the sample mean from these response times, denoted by $\hat{r}$, was used to estimate mean response time r of an FCFS $G/G/1$ queueing system. We further construct new confidence intervals of r for a $G/G/1$ queueing system, which are based on four bootstrap methods; standard bootstrap (SB) confidence interval, percentile bootstrap (PB) confidence interval, bias-corrected percentile bootstrap (BCPB) confidence interval, and bias-corrected and accelerated (BCa) confidence interval. A numerical simulation study is conducted in order to demonstrate performance of the proposed estimator $\hat{r}$ and bootstrap confidence intervals of r. From the simulation results, we show that $\hat{r}$ is a consistent estimator of r. In addition, we also investigate the accuracy of the four bootstrap confidence intervals by calculating the coverage percentage, the average length, and the relative average length of confidence intervals. Detailed discussions of simulation results for seven various queueing models are presented.

Keywords: Bootstrap confidence interval; Coverage percentage; $G/G/1$ queue; Mean response time; Recursion relation

1. Introduction

In the analysis of queueing theory, the response time plays an important index in measuring the efficient characteristic of a system. The response time R is defined as the time spent by a customer from arrival until departure; R also can be viewed as the time elapsed from the instant of job arrival until its completion. Modern is a competitive time. To everyone, time is money. We often require the service time of each queue as short as possible. Thus the mean response time $E(R)$, denoted by r, is an important index for a customer to determine whether entering a queueing service system. This paper deals with the estimation and confidence interval of mean response time for a $G/G/1$ queueing system using data-based recursion relation and bootstrap methods.

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The statistical inference in queueing problems are rarely found in the literature and the work of related problems in the past mainly concentrates on only *parametric statistical inference*, in which the distribution of population is with a known form (except perhaps for the parameters). The pioneering paper in parameter estimation problem was first proposed by Clarke [1], who developed maximum likelihood estimates for the arrival and service parameters of an $M/M/1$ queue. Lilliefors [2] examined the confidence intervals for the $M/M/1$, $M/E_k/1$ and $M/M/2$ queues. For a $G/G/1$ queue, Basawa and Prabhu [3] studied moment estimates as well as maximum likelihood estimates. Maximum likelihood estimates and confidence intervals in an $M/M/2$ queue with heterogeneous servers were derived by Dave and Shah [4] and Jain and Templeton [5], respectively. The confidence interval estimation of a single server queue with random arrivals and balking was investigated by Rubin and Robson [6]. The hypothesis testing and simultaneous confidence intervals of the system parameters for an $M/E_k/1$ queue were analyzed by Jain [7]. Abou-El-Ata and Hariri [8] developed point estimation and confidence intervals of the truncated $M/M/2/N$ queue with balking and heterogeneous servers. Basawa et al. [9] used the waiting time data to deal with the maximum likelihood estimation for a single server queue. An overview of literature on the statistical analysis of several queueing systems was provided by Dshalalow [10]. Rodrigues and Leite [11] applied Bayesian analysis to study confidence intervals of an $M/M/1$ queue. Huang and Brill [12] derived the minimum variance unbiased estimator and the maximum likelihood estimator of a collection of n independent $M/G/c/c$ queues. Recently, Wang et al. [13] examined the maximum likelihood estimates and confidence intervals of an $M/M/R/N$ queue with balking and heterogeneous servers. Existing research works, including those mentioned above, most focused on the *parametric statistical inferences* of arrival rate, service rate and expected number of customers in the system. So far very few authors have studied the *nonparametric statistical inferences* of the mean response time. Chu and Ke [14] examined the statistical behavior of the mean response time for the $M/G/1$ queueing system using bootstrapping simulation. However, the functional forms of the interarrival time and service time are seldom known. This motivates us to develop the nonparametric simulated methods of estimating mean response time r for a $G/G/1$ queueing system under FCFS discipline.

This paper is aimed to investigate estimation of r through some efficient and robust approaches. Firstly in Section 2, we deduce the point estimator $\hat{r}$ of r by a data-based recursion relation when the distributions of interarrival and service times of the underlying FCFS queueing system are unknown. Next, we use the bootstrap method to construct new confidence intervals of r in Section 3. We sketch the bootstrap estimation of r, and then the four types of bootstrap confidence intervals for r are presented in the order of SB, PB, BCPB, and BCa. Subsequently, Section 4 is devoted to evaluate the performance of the estimator $\hat{r}$ and the four bootstrap confidence intervals for r in terms of simulation analysis. Following simulation studies from seven different queueing models, we find that $\hat{r}$ is a consistent estimator of r. Moreover, we assess and compare the accuracy of these four bootstrap interval estimation methods by virtue of coverage percentage and relative average length of intervals. Finally, some discussions are carried out and some conclusions are drawn.

2. Estimation of mean response time

In practical queueing system, the distributions of interarrival and service times are unknown. It is difficult to be obtained the exactly theoretic value of $r = E[R]$ for a $G/G/1$ queueing model (see [15]). In this section, we present an estimation approach of r by means of using a data-based recursion relation.

Let $X_1, X_2, \ldots, X_n$ be a random sample from a continuous random variable X, and let $Y_1, Y_2, \ldots, Y_n$ be a random sample from a continuous random variable Y that is independent of X. Let (X_i, Y_i) represent interarrival time and service time for the ith customer of an FCFS queueing system. Also we use R_i to represent the response time of the ith customer entering the FCFS queueing system. Intuitively, we know that $(R_1, R_2, \ldots, R_n)$ are determined from $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$. Denote W_i by the waiting time of the ith customer of the FCFS queueing system, then

$$R_i = W_i + Y_i \tag{1}$$

for $i = 1, 2, \ldots, n$.

Similar to the analysis by Kleinrock [15, Chapter 9], and referencing Fig. 1, we can easily evaluate $W_1, W_2, \ldots, W_n$ using a simple recursion relation given by