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A new nonlinear neural network for solving a class of constrained parametric optimization problems

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Abstract

The paper deals with convex parametric programming problems. In this paper convex parametric programming transform to a neural network model and then we solve neural network model with one of numerical methods. Finally, simple numerical examples are provided for the sake of illustration.

Keywords: Convex parametric programming; Neural networks; Stability

1. Introduction

Optimization problems arise in a wide variety of scientific and engineering applications including signal processing, system identification, filter design, function approximation, regression analysis, and so on. In many engineering and scientific applications, the real-time solution of optimization problems is widely required. However, traditional algorithms for digital computers may not be efficient since the computing time required for a solution is greatly dependent on the dimension and structure of the problems. One possible and very promising approach to real-time optimization is to apply artificial neural networks. Because of the inherent massive parallelism, the neural network approach can solve optimization problems in running time at the orders of magnitude much faster than those of the most popular optimization algorithms executed on general-purpose digital computers.

Parametric programming models appear in mathematics, engineering, physics and other sciences when some processes or systems depend on a parameter which this parameter can be change in a interval. Many optimization problems are naturally cast as parametric problems; for instance, continuous time, optimal control problems subject to all time state constraints. In this paper, we design neural network and apply to solve parametric programming problems. We show how linear and nonlinear parametric programming by using of method told, will transform to a neural network model.

In 1985 and 1986 Hopfield and Tank [1,2] proposed a neural network for solving linear programming problems. Their seminal work has inspired many researchers to investigate alternative neural networks for solving

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linear and nonlinear programming problems. In 1987, Kennedy and Chua [3] proposed an improved model that always guaranteed convergence. However, their new model converges to only an approximation of the optimal solution. In 1990, Rodriguez-Vazquez et al. [4] proposed a class of neural networks for solving optimization problems. In 1996, Wu et al. [5] introduced a new model that solves both the primal and dual problems of linear and quadratic programming problems. Their new model always globally converge to the solutions of the primal and dual problems. In 2002, Xia et al. [6] introduced a recurrent neural network for solving the nonlinear projection formulation. In 2004, Effati et al. [7] presented a new nonlinear neural network that has a much faster convergence. Their new model is based on a nonlinear dynamical system.

2. Convex parametric programming problems

Let us consider the following parametric programming problem:

$$\begin{aligned} &\text{minimize} \quad f(x,\lambda) \\ &\text{subject to} \\ &\qquad h_i(x,\lambda) \leqslant 0, \quad i=1,\ldots,m \\ &\qquad \lambda \in [\alpha,\beta], \quad \alpha \quad \text{and} \quad \beta \text{ are constant.} \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $f(x,\lambda)$ and $h_i(x,\lambda)(i=1,\ldots,m)$ are convex functions with respect to the first argument. It is also assumed that f, $h_i(i=1,\ldots,m)$ are twice continuously differentiable, and for each $\lambda \in [\alpha,\beta]$ problem (1) has feasible solution.

Now problem (1) transform to a neural network model. In general, if $f(x,\lambda)$ is nonlinear and if the penalty method is applied to solve (1), then we can obtain an unconstrained optimization problem:

$$\min_x P(x,\lambda) = f(x,\lambda) + \frac{k}{2}\sum_{i=1}^{m}\left(h_i^+(x,\lambda)\right)^2, \tag{2}$$

where k is a positive number and

$$h_i^+(x,\lambda) = \max_x\{0, h_i(x,\lambda)\} \quad (i=1,\ldots,m),\ \lambda \in [\alpha,\beta] \text{ is fixed.}$$

Thus, the necessary condition for optimality of (2) for each $\lambda \in [\alpha,\beta]$ is:

$$\frac{\partial P(x,\lambda)}{\partial x} = \frac{\partial f(x,\lambda)}{\partial x} + k\sum_{i=1}^{m} h_i^+(x,\lambda)\frac{\partial h_i(x,\lambda)}{\partial x} = 0. \tag{3}$$

We define the following neural network model:

$$x'(t,\lambda) = -\frac{\partial f(x(t),\lambda)}{\partial x} - k\sum_{i=1}^{m} h_i^+(x(t),\lambda)\frac{\partial h_i(x(t),\lambda)}{\partial x}, \tag{4}$$

where

$$\frac{\partial f(x,\lambda)}{\partial x} = \left(\frac{\partial f}{\partial x_1},\frac{\partial f}{\partial x_2},\ldots,\frac{\partial f}{\partial x_n}\right)^{\mathrm{T}}$$

and

$$\frac{\partial h_i(x,\lambda)}{\partial x} = \left(\frac{\partial h_i}{\partial x_1},\frac{\partial h_i}{\partial x_2},\ldots,\frac{\partial h_i}{\partial x_n}\right)^{\mathrm{T}}.$$

Proposition 1. *If for any k (2) has an optimal solution, and if for system (4) we can find a state variable $x(t,\lambda)$ such that the neural network (4) is asymptotically stable at $x^*(\lambda)$, then the optimal solution to (2) will be the equilibrium state of (4).*