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The dragon war

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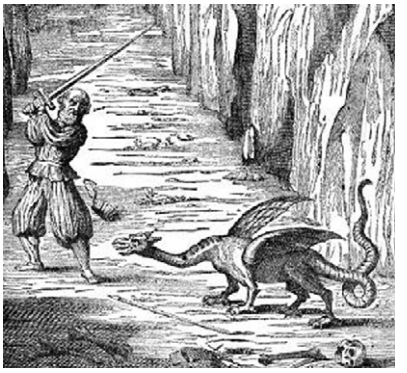
Abstract

This article presents an approach to solve the travelling salesman problem using linear optimisation with a suitable set of constraints. Such approaches are well known as Cutting Plane methods but previously applied algorithms still suffer from the NP-completeness of the problem causing an exponential number of computational steps in the worst case. The method presented in this paper both tolerates non-integer results for a dedicated class of solutions (symmetric solutions) and avoids other ones (asymmetric solutions). It is thus able to deal with the inherent complexity of the travelling salesman problem.Therefore the overall complexity of the optimisation algorithm remains polynomial in the worst case.

Keywords: Travelling salesman problem; Discrete optimisation; Cutting plane optimisation; NP-completeness; P/NP problem

1. Introduction

There was once upon a time a prospering and powerful country, the great kingdom of Informatia. Large data fields were supplied by high-capacity channels and broad message streams pervaded the wide lands. The able and enterprising citizens of Informatia created wonderful works of art, algorithms and networks and the builders and designers spared no effort to erect splendid software architectures.



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There was only one thing that saddens the Informatians: The dragons. Nobody knew when they had occurred for the first time, nevertheless they were there, settling everywhere along the boundaries of Polynomia, the large region right in the middle of Informatia. The dragons controlled all the gates and nobody could pass them without losing his precious MIPS and FLOPS. So, people could not get through and the vast territories neighbouring to Polynomia (the NP-territories) were deserted and desolate.

During a long lasting and exhaustive fight many brave warriors and mighty wizards tried to defeat or at least to repel the beasts, but all their effort failed: The power of the dragons could not be broken. Finally, the wisest scientists of Informatia came together for an extensive investigation of their enemies and at the end they made a horrible discovery: The worst of the beasts (which had been called completely non-passable) were magically coupled together combining all their power. Whoever would try to go through one single gate had to overwhelm the joined might of thousands of dragons. Faced with this grisly monstrosity, the Informatians became desperate and started to accept that they were doomed until the end of the days: Polynomia was a huge prison.

Hundreds of years later, Merlin was a young wizard in his early 120s living quite near to a gate which was guarded by one of the NP-complete beasts. Preferentially attacking business people this dragon had been called the travelling salesman problem. By accident Merlin had once got in his way during a trip into the mountains and after a short and unsuccessful resistance he had to run for his life. Enraged about this shaming indignity, Merlin decided to get rid of the problem.

The description of travelling salesman problem (TSP) is quite simple: Given a set of n station and distances c_{ab} for each disjoint pair a, b of stations find a roundtrip of minimal total length visiting each station exactly once. In the symmetric TSP is $c_{ab} = c_{ba}$, in the asymmetric case c_{ab} may be different from c_{ba}.

In spite of the plainness of the problem definition the TSP is a representative of the NP-complete problems which are known as the most difficult members of the complexity class NP: Solutions for such problems require an exponential number of computational steps on a deterministic machine and until now there is no general applicable algorithm which is able to solve a NP-complete problem with a guaranteed polynomial number of steps [1,2].

2. Approach

After observing TSP for some time Merlin realised that he could not dare a fight without aid. So he decided to get assistance by the SimpleX. This was a helpful and very powerful creature and nothing in the world could resist him, not even a dragon. There was just one problem with the SimpleX: It was not only extremely strong but also extremely simple minded (that is where he got his name from) so you could not let him act self-dependent at all. In fact, you had to dictate him any single step and any little action even of a simple work.

This had to be done by bars of unbreakable material, the linear constraints. Merlin's intention was to build up a solid constraint cage around TSP and SimpleX such that the dragon could not escape and SimpleX could crush him. This sounds quite easy but you have to consider that the constraint bars were made of a very rare and expensive material so you had to get by with a reasonable number of it (a polynomial number in the Informatian's way of speaking). Nevertheless, Merlin was of good cheer, so he took some constraints and started to construct a cage.

The approach presented in this paper solves the asymmetric TSP using linear optimisation with an appropriate set of constraints. The basic idea is to treat the model parameters like real values though the TSP is an integer optimisation problem. Such approaches are well-known as Cutting Plane Algorithms and have shown some amazing results [3,4]. Nevertheless, the computational effort of these approaches is still exponential in the worst case.

Generally, a linear optimisation requires the definition of a cost function

$$\text{Min} \sum_{i=1}^{p} c_i x_i,$$