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Oscillation criteria for second order differential equations with positive and negative coefficients

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Abstract

Some oscillation criteria for the second order neutral delay differential equations

$$\left[x(t) \pm \sum_{i=1}^{l} c_i(t)x(t-\tau_i)\right]'' + \sum_{i=1}^{m} p_i(t)x(t-\delta_i) - \sum_{i=1}^{n} q_i(t)x(t-\sigma_i) = 0, \quad t > 0$$

are established. New oscillation criteria are different from one recently established in the sense that the boundedness of the solution in the results of Parhi and Chand [Oscillation of second order neutral delay differential equations with positive and negative coefficients, J. Indian Math. Soc., 66 (1999) 227–235.] has been erased., i.e. we give sufficient conditions for the oscillation of all solutions.

Keywords: Oscillation criteria; Second order; Positive and negative coefficients

1. Introduction

In this paper we consider the oscillation of the second order neutral delay differential equations

$$\left[x(t) + \sum_{i=1}^{l} c_i(t)x(t-\tau_i)\right]'' + \sum_{i=1}^{m} p_i(t)x(t-\delta_i) - \sum_{i=1}^{n} q_i(t)x(t-\sigma_i) = 0, \quad t > 0, \tag{E_1}$$

$$\left[x(t) - \sum_{i=1}^{l} c_i(t)x(t-\tau_i)\right]'' + \sum_{i=1}^{m} p_i(t)x(t-\delta_i) - \sum_{i=1}^{n} q_i(t)x(t-\sigma_i) = 0, \quad t > 0, \tag{E_2}$$

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where $m \geqslant n$, τ_i $(i = 1,2,\ldots,l)$, δ_i $(i = 1,2,\ldots,m)$, σ_i $(i = 1,2,\ldots,n)$ are nonnegative constants, $\delta_i \geqslant \sigma_i$ $(i = 1,2,\ldots,n)$ and

$$\begin{aligned} &c_i(t) \in C([0,\infty);[0,\infty)) \quad (i = 1,2,\ldots,l);\\ &p_i(t) \in C([0,\infty);[0,\infty)) \quad (i = 1,2,\ldots,m);\\ &q_i(t) \in C([0,\infty);[0,\infty)) \quad (i = 1,2,\ldots,n). \end{aligned}$$

By a solution of (E_1) (or (E_2)) we mean a continuous function $x(t)$ which is defined for $t \geqslant t_0 - T$, such that $x(t) + \sum_{i=1}^{l} c_i(t)x(t-\tau_i) \in C^2([t_0,\infty);\mathbb{R})$ and (E_1) (or (E_2)) is satisfied identically for all $t \geqslant t_0$, where

$$T = \max\{\tau_i, \delta_j, \sigma_k : \ 1 \leqslant i \leqslant l, \ 1 \leqslant j \leqslant m, \ 1 \leqslant k \leqslant n\}.$$

We restrict our attention only to the nontrivial solution $x(t)$, i.e. to the solution $x(t)$ such that $\sup\{|x(t)| : t \geqslant t_1\} > 0$ for all $t_1 \geqslant t_0$. A nontrivial solution of (E_1) (or (E_2)) is called *oscillatory* if it has arbitrary large zeros, otherwise, it is called nonoscillatory. The equation is called oscillatory if all its solutions are oscillatory.

Sufficient conditions for oscillation of solutions of first order neutral delay differential equations with positive and negative coefficients have been investigated by many authors [1–3,5–8]. On the other hand, the recent paper by Parhi and Chand [4] contains various sufficient conditions for the oscillation of all *bounded* solutions of the second order neutral delay differential equations (E_1) and (E_2). Naturally, the question arises: Is it possible to establish the oscillation criteria for all solutions of this equations and to erase rather restrictive condition of boundedness of solutions. The object of this paper is to give the affirmative answer to this question and to establish sufficient conditions for the oscillation of all solutions of this equations.

Hereafter, we always assume without mentioning that

$$\begin{cases} q_i(t) \leqslant q_i(t-\sigma_i) & (i = 1,2,\ldots,n),\\ p_i(t) \geqslant q_i(t-\delta_i) & (i = 1,2,\ldots,n); \end{cases} \tag{H$_1$}$$

$$p_j(t) - q_j(t-\delta_j) \geqslant k_j > 0 \text{ for some } j \in \{1,2,\ldots,n\} \text{ and some } k_j > 0. \tag{H$_2$}$$

The paper is organized as follows. In Section 2, we establish the sufficient conditions for the oscillation of all solutions of Eq. (E_1), while in Section 3 we deals with Eq. (E_2). Oscillation results of nonhomogeneous cases of (E_1) and (E_2) are given in Section 4. In every Section we are going to give some examples illustrating obtained results.

2. Oscillatory behavior of solutions of Eq. (E_1)

In this section we obtain the following oscillation criteria for Eq. (E_1).

Theorem 1. *Assume that*

$$0 \leqslant \sum_{i=1}^{l} c_i(t) \leqslant c, \quad c = \text{const.} \tag{H$_3$}$$

Eq. (E_1) *is oscillatory, if*

$$\sum_{i=1}^{n} \int_0^\infty \int_{s-\delta_i}^{s-\sigma_i} q_i(\xi)\,\mathrm{d}\xi\,\mathrm{d}s \leqslant 1. \tag{C$_1$}$$

Proof. Suppose that $x(t)$ is a nonoscillatory solution of (E_1). Without any loss of generality, we assume that $x(t) > 0$ for $t \geqslant t_0$. If we define

$$z(t) = x(t) + \sum_{i=1}^{l} c_i(t)x(t-\tau_i) - \sum_{i=1}^{n} \int_{t_0}^{t} \int_{s-\delta_i}^{s-\sigma_i} q_i(\xi)x(\xi)\,\mathrm{d}\xi\,\mathrm{d}s, \tag{1}$$