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Inconsistent fuzzy linear systems ☆

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Abstract

The $m \times n$ inconsistent fuzzy linear system is studied. The fuzzy least squares solution and therefore the weak fuzzy least squares solution to the fuzzy system are expressed by using the generalized inverses of the coefficient matrix. The existence of the strong fuzzy least squares solution to the fuzzy system is also discussed. Some examples are presented to illustrate the theory.

Keywords: Inconsistent; Fuzzy linear system; Fuzzy least squares solution; Generalized inverse; *Moore–Penrose* inverse; Nonnegative matrix

1. Introduction

Friedman et al. [1] proposed a general model for solving an $n \times n$ fuzzy linear system, whose coefficients matrix is crisp and right-hand side column is an arbitrary fuzzy number vector, by the embedding approach. Asady et al. [2], who merely discuss the full row rank system, use the same method to solve the $m \times n$ fuzzy linear system for $m \leqslant n$. Zheng and Wang [3] discuss the solution of the general $m \times n$ consistent fuzzy linear system.

In this paper, we investigate the $m \times n$ inconsistent fuzzy linear system whose coefficients matrix is crisp and right-hand side column is a fuzzy number vector. Similar to that in [1–3], we first replace the original $m \times n$ fuzzy linear system by a $(2m) \times (2n)$ crisp function linear system. And then the fuzzy least squares solutions to the system are discussed by using the generalized inverses of the coefficients matrix.

In Section 2 we recall the preliminaries for the $m \times n$ fuzzy linear system and the general model for solving the system. The solutions to the fuzzy linear system are discussed in Section 3 and numerical examples are given to illustrate our theory in Section 4.

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2. Preliminaries

An arbitrary fuzzy number is represented, in parametric form, by an ordered pair of functions $(\underline{u}(r), \bar{u}(r)),\ 0 \leqslant r \leqslant 1$, which satisfy the following requirements [4]:

1. $\underline{u}(r)$ is a bounded left continuous nondecreasing function over $[0,1]$.
2. $\bar{u}(r)$ is a bounded left continuous nonincreasing function over $[0,1]$.
3. $\underline{u}(r) \leqslant \bar{u}(r),\ 0 \leqslant r \leqslant 1$.

A crisp number α can be simply expressed as $\underline{u}(r) = \bar{u}(r) = \alpha,\ 0 \leqslant r \leqslant 1$.

The addition and scalar multiplication of fuzzy numbers previously defined can be described as follows, for arbitrary $u = (\underline{u}(r), \bar{u}(r)),\ v = (\underline{v}(r), \bar{v}(r))$ and real number λ,

(a) $u + v = (\underline{u}(r) + \underline{v}(r), \bar{u}(r) + \bar{v}(r))$;

(b) $\lambda u = \begin{cases} (\lambda\underline{u}(r), \lambda\bar{u}(r)), & \lambda \geqslant 0, \\ (\lambda\bar{u}(r), \lambda\underline{u}(r)), & \lambda < 0. \end{cases}$

Definition 2.1. The $m \times n$ linear system

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = y_1, \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = y_2, \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = y_m, \end{cases} \tag{2.1}$$

where the coefficients matrix $A = (a_{ij})$ is a crisp $m \times n$ matrix and y_i, $i = 1, 2, \ldots, m$ are fuzzy numbers, is called a fuzzy linear system (FLS).

Let $x_j = (\underline{x}_j(r), \bar{x}_j(r)),\ j = 1, 2, \ldots, n$ and $y_i = (\underline{y}_i(r), \bar{y}_i(r)),\ i = 1, 2, \ldots, m$ be fuzzy numbers. Then FLS (2.1) can be represented in the form of the following function linear system:

$$\begin{cases} \underline{\sum_{j=1}^{n} a_{ij}x_j} = \sum_{j=1}^{n} \underline{a_{ij}x_j} = \underline{y}_i, \\ \overline{\sum_{j=1}^{n} a_{ij}x_j} = \sum_{j=1}^{n} \overline{a_{ij}x_j} = \bar{y}_i, \end{cases} \quad i = 1, 2, \ldots, m. \tag{2.2}$$

In particular, if $a_{ij} \geqslant 0$, $1 \leqslant j \leqslant n$ for some i, then

$$\sum_{j=1}^{n} a_{ij}\underline{x}_j = \underline{y}_i, \qquad \sum_{j=1}^{n} a_{ij}\bar{x}_j = \bar{y}_i. \tag{2.3}$$

Definition 2.2. A fuzzy number vector $(x_1, x_2, \ldots, x_n)$ given by

$$x_i = (\underline{x}_i(r), \bar{x}_i(r)), \quad 1 \leqslant i \leqslant n,\ 0 \leqslant r \leqslant 1$$

is called a solution of the fuzzy system if it satisfies (2.2).

To solve (2.1), following [1], we may assume a $(2m) \times (2n)$ matrix $S = (s_{ij})$ is determined as follows:

$$\begin{aligned} &a_{ij} \geqslant 0 \Rightarrow s_{ij} = a_{ij}, \quad s_{m+i,n+j} = a_{ij}, \\ &a_{ij} < 0 \Rightarrow s_{i,n+j} = -a_{ij}, \quad s_{m+i,j} = -a_{ij}, \end{aligned} \tag{2.4}$$

and any s_{ij} which is not determined by (2.4) is zero. In this case, the system (2.2) is extended to the following crisp block form