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Computational and theoretical pitfalls in some current performance measurement techniques; and a new approach

M. Soleimani-damaneh [a,*], G.R. Jahanshahloo [a], S. Abbasbandy [b]

[a] *Department of Mathematics, Teacher Training University, 599 Taleghani Avenue, Tehran 15618, Iran*
[b] *Department of Mathematics, Imam-Khomeini University, Qazvin, Iran*

Abstract

This paper addresses some pitfalls of some fuzzy data envelopment analysis models which have been provided in recent published papers. This study deals with these papers from both computational and theoretical points of view. Moreover, a new approach to deal with fuzzy data in DEA framework is provided, which does not have the mentioned pitfalls.

Keywords: Linear programming; Duality; Simplex algorithm; Data envelopment analysis; Fuzzy data envelopment analysis

1. Introduction

Data envelopment analysis (DEA) is a non-parametric technique for measuring the relative efficiencies of a set of decision making units (DMUs) which consume multiple inputs to produce multiple outputs. This technique was initially proposed by Charnes et al. (CCR model) [4] and was improved by other scholars, especially Banker et al. (BCC model) [1]. Nowadays, DEA has allocated to itself a wide variety of applied research. There is a great variety of research dealing with the evaluation of DMUs with crisp data; see, e.g., [5,17].

In the real world a production system usually involves fuzzy factors, which leads to fuzzy data. This necessitates an approach which is able to deal with fuzzy data. One can find several fuzzy mathematical programming-based approaches to assessing DMUs in fuzzy DEA literature; see, e.g., [7–14]. Here, we focus on some recent published fuzzy DEA papers and deal with two questions for each provided technique (model):

(i) Is the provided approach a general way to capture all types of fuzzy data using all fuzzy DEA models?
(ii) Is the provided approach efficient from a computational point of view?

* Corresponding author.
E-mail address: soleimani_d@yahoo.com (M. Soleimani-damaneh).

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Regarding the above-mentioned questions, we address some pitfalls existing in some recent published fuzzy DEA papers. Moreover, a new approach to deal with fuzzy data in DEA framework is provided, which does not have the stated pitfalls.

The rest of this paper is organized as follows: Section 2 reviews some basic concepts about fuzzy numbers and their ranking. In Sections 3 and 4, we survey questions (i) and (ii) for some fuzzy DEA models, respectively, and point out some pitfalls. Section 5 introduces a new fuzzy DEA methodology and explains the properties of this approach. Finally, Section 6 contains some conclusions.

2. Fuzzy numbers and ranking

A fuzzy set on a set X is a classical function $\tilde{a} : X \to [0,1]$. The support of $\tilde{a}$, $\mathrm{supp}\,\tilde{a}$, is the closure of the set $\{x \in X | \tilde{a}(x) > 0\}$. A fuzzy number is a fuzzy set $\tilde{a} : \mathbb{R} \to [0,1]$ on $\mathbb{R}$, satisfying three conditions: (a) $\tilde{a}$ is an upper semi continuous function on $\mathbb{R}$, (b) $\mathrm{supp}\,\tilde{a}$ is a compact interval, and (c) if $\mathrm{supp}\,\tilde{a} = [a, b]$, then there exist c, d, such that $a \leqslant c \leqslant d \leqslant b$ and $\tilde{a}$ is increasing on the interval $[a,c]$, equal to 1 on the interval $[c,d]$, and decreasing on the interval $[d,b]$. The α-cut set of $\tilde{a}$, denoted by $[\tilde{a}]_\alpha$, is

$$[\tilde{a}]_\alpha = \{x \in \mathbb{R} | \tilde{a}(x) \geqslant \alpha\} \tag{1}$$

for each $\alpha \in (0,1]$, while $[\tilde{a}]_0 = \mathrm{supp}\,\tilde{a}$. The lower and upper endpoints of any α-cut set, $[\tilde{a}]_\alpha$, are represented by $[\tilde{a}]_\alpha^{\mathrm{L}}$ and $[\tilde{a}]_\alpha^{\mathrm{U}}$, respectively. Also in this paper, let $\mathbb{F}(\mathbb{R})$ be the family of fuzzy numbers on $\mathbb{R}$.

Literature review reveals that multitudes of fuzzy number ranking methods exist. Papers by Bortolan and Degani [3] as well as Wang and Kerre [19,20] present a comprehensive survey of the available methods. Some methods use the concept of mean value for this task [6,15]. Some others compare the fuzzy numbers using the concept of α-cut sets [8]. There are some possibilistic rankings of fuzzy numbers, e.g., [10]. Also, there is another class of ranking methods called distance-based approaches (see [16,18,21]). Here we consider one approach in this class. Saade and Schwarzlander [16] used interval to consider ordering. They only used non-negative values to compare the ordering of fuzzy numbers and do not use the concept of sign. Yao and Wu [21] consider the signed distance d^* on $\mathbb{R}$ such that $d^*(a,0) = a$ and $d^*(a,b) = a - b$ for all $a, b \in \mathbb{R}$. Then for $\tilde{a}, \tilde{b} \in \mathbb{F}(\mathbb{R})$, they define the signed distance as

$$d(\tilde{a},\tilde{b}) = 1/2 \int_0^1 \left([\tilde{a}]_\alpha^{\mathrm{L}} + [\tilde{a}]_\alpha^{\mathrm{U}} - [\tilde{b}]_\alpha^{\mathrm{L}} - [\tilde{b}]_\alpha^{\mathrm{U}}\right) \mathrm{d}\alpha. \tag{2}$$

They prove that d is an extension of d^* and they define a ranking system on $F(\mathbb{R})$ as

$$\tilde{b} \prec \tilde{a} \text{ if } d(\tilde{a},\tilde{b}) > 0, \quad \tilde{b} \succ \tilde{a} \text{ if } d(\tilde{a},\tilde{b}) < 0, \quad \text{and} \quad \tilde{a} \approx \tilde{b} \text{ if } d(\tilde{a},\tilde{b}) = 0. \tag{3}$$

The reasonable properties of this ranking method have been proved in [21]. In this paper we use this ranking approach to provide a fuzzy DEA model.

An LR-fuzzy number $\tilde{a}$ can be described with the following membership function:

$$\tilde{a}(x) = \begin{cases} L\left(\frac{\underline{m}-x}{\beta}\right), & \underline{m} - \beta \leqslant x \leqslant \underline{m}, \\ 1, & \underline{m} \leqslant x \leqslant \overline{m}, \\ R\left(\frac{x-\overline{m}}{\gamma}\right), & \overline{m} \leqslant x \leqslant \overline{m} + \gamma, \\ 0, & \text{otherwise,} \end{cases} \tag{4}$$

where $L, R : [0,1] \to [0,1]$, with $L(0) = R(0) = 1$ and $L(1) = R(1) = 0$, are non-increasing, continuous shape functions. The LR-fuzzy number is then denoted by $\tilde{a} = (\underline{m}, \overline{m}, \beta, \gamma)_{LR}$, and $[\underline{m}, \overline{m}]$ is the peak of $\tilde{a}$. The α-cut sets of $\tilde{a}$ can easily be computed as

$$[\tilde{a}]_\alpha = [\underline{m} - L^{-1}(\alpha)\beta, \overline{m} + R^{-1}(\alpha)\gamma], \quad \alpha \in [0,1]. \tag{5}$$