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A bypassing path based routing algorithm for the pyramid structures ☆

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Abstract

A pyramid structure is a collection of connected mesh structures. It is well known that a pyramid structure of size n has a diameter of $\log n$, which immediately leads to a natural routing algorithm, using only links between adjacent meshes. However, such paths are often unavailable because of the existence of faulty nodes and/or links. In this paper, we present the concept and technical details of an alternative routing algorithm, based on a collection of bypassing paths, for the potentially faulty pyramid structures that makes use of the links both within and between meshes contained in such structures.

We also study some of the mathematical properties of this set of bypassing paths, including their average lengths, to suggest a characterization of the proposed alternative routing algorithm.

Keywords: Routing algorithms; Pyramid structures; Performance evaluation

1. Introduction

Besides those well-known interconnection structures such as bus, ring, tree, hypercube and its various cousins [14], the pyramid structure has also been extensively studied as a model for many parallel processing activities, particularly, in supporting various graph algorithms such as component labeling, image shrinking and expanding, identifying minimum spanning tree, etc.; and in solving various computational geometric problems such as studying convexity, solving nearest point problems, etc., [17,6,13,14,18,8].

As a basic support for any parallel algorithm based on such a distributed structure, an effective and efficient routing mechanism must be provided so that information can be exchanged between computing nodes. Modern parallel architectures, assuming no node/link contention and short message, often adopt the wormhole model [9], with the main benefit being that, as long as the aforementioned assumptions hold, the time it takes to transmit messages is independent of the distance between the two involved nodes. But, in this work, we explicitly assume the existence of faulty nodes(links) which necessarily leads to node/link contention. Thus, the data transmission we study in this work does depend on the traveling distance of the routed data. As a result, the problem of measuring the length of the paths as generated by routing algorithms becomes interesting.

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To meet the needs of such QoS requirements as guaranteed minimum bandwidth and maximum end-to-end delay, it is only natural for the routing algorithm to look for a "widest and shortest path", namely, a path with maximum residual capacity, and minimum hop count, between the involved nodes, which is almost always the approach any routing algorithm, particularly a minimal one, follows [7]. Thus, when characterizing the global behavior of the relevant routing algorithms, or more generally, that of the involved network structure, people often use such measurement as the *diameter* of the network, defined as *the maximum distance between any pair of processors in the network* [3]. It is noted that a significant advantage of using a pyramid structure of size n,[1] referred to as $P(n)$ throughout this paper, as the interconnection network is that its diameter is only in the order of $\log n$, and the corresponding communication path, henceforth, the *ideal path,* which makes use of only the parent–child links[2] that connects the sending node and the receiving one, is naturally adopted to implement a routing algorithm.

However, since the notion of the diameter corresponds to a *shortest path* between two furthest apart processors in the network, it is certainly not a surprise that the diameter of a structure "is sometimes an overly optimistic lower bound for certain problems and machine models" [18, p. 5]. This is certainly true for the pyramid structure as well: when a lot of data movement is involved, the *apex*[3] of such a structure turns into a bottleneck. Moreover, any node along the ideal path between two involved nodes could be potentially faulty [8]. As a result, the corresponding ideal path is often not available. On the other hand, those links contained in various levels of meshes could also be used to send messages. Indeed, as pointed out in [18, p. 282], those "nearest point connections may be used at the intermediate levels to circumvent this bottleneck." and, in the same paper, a whole collection of pyramid based SIMD algorithms are presented, solving various graph theory related problems, such as component labeling, minimum spanning forests, etc., that make use of links both between, and within, the involved meshes. In particular, as a basic support for these parallel algorithms, Miller et al. discussed a mechanism [17,18] that moves data from one level of a fault-free pyramid structure to another. More specifically, a packet is broadcast within the mesh, where the sending node is located, in a fixed snake-like, or row-major, ordering; and during this broadcasting process, every node sends the packet up(down) to the next level using the parent–child links.

Hsieh et al. in [11] put forward an O(1) shortest path routing algorithm, again for the fault-free pyramids, based on a condition that decides which path between two nodes is shorter: the one consisting of only mesh links, or the one consisting of parent–child links together with mesh links connecting the parents of the involved nodes in the layer above.

In dealing with the fault tolerance situation, Cao et al. in [8] discussed the fault tolerance properties of the pyramid structures. In particular, they showed that the line (node) connectivity of the pyramid structures is 3, and presented an algorithm that constructs a path in $P(n)$ with two faulty nodes, and an $O(m^2)$, $m = \log_4(n)$, algorithm in $P(n)$ with no faulty nodes for constructing a set of three edge disjoint paths with the length of the longest one no more than $\frac{10}{3}m + 6$.

Chen et al. also presented, in [5], a $O(n)$ fault-tolerant routing algorithm for $P(n)$, following the least level minimal approach which prefers those routing paths located at the lowest level. Their algorithm allows at most three faulty nodes and/or edges in the pyramid structure, and takes at most $4n - 2$ steps to send over a packet between a pair of nodes in $P(n)$.

In this paper, based on a collection of the so-called *bypassing paths,* we propose an alternative routing algorithm, a randomized version and then a deterministic one, for potentially faulty pyramid structures, where multiple, no more than $\log_2 n - 3$, faulty nodes and/or the links in between may occur along the aforementioned ideal path.[4] We also rigorously analyze some of the mathematical properties of those bypassing paths and show that, in the extreme case where the two involved nodes are furthest apart, the length of the shortest

[1] A pyramid structure actually has $\frac{1}{3}(4n - 1)$ nodes. But, traditionally, such a structure is said to have size n [18], which is really the size of its base mesh.

[2] Intuitively, a parent–child link refers to a connection between two nodes adjacent in two neighboring meshes. A precise definition of the pyramid structure, and the associated concepts such as parent–child links and apex node, are forthcoming in the following Section 2.

[3] Intuitively, the apex of a pyramid is its top, or root, node.

[4] Faulty nodes may also exist within the mesh structures. But, as discussed in Section 7, their locations turn out to be critical for the successful construction of the routing paths, particularly if shortest routing paths are sought.