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Extension of the TOPSIS method for decision-making problems with fuzzy data

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Abstract

Decision making problem is the process of finding the best option from all of the feasible alternatives. In this paper, from among multicriteria models in making complex decisions and multiple attribute models for the most preferable choice, technique for order preference by similarity to ideal solution (TOPSIS) approach has been dealt with. In real-word situation, because of incomplete or non-obtainable information, the data (attributes) are often not so deterministic, there for they usually are fuzzy/imprecise. Therefore, the aim of this paper is to extend the TOPSIS method to decision-making problems with fuzzy data. In this paper, the rating of each alternative and the weight of each criterion are expressed in triangular fuzzy numbers. The normalized fuzzy numbers is calculated by using the concept of *α-cuts*. Finally, a numerical experiment is used to illustrate the procedure of the proposed approach at the end of this paper.

Keywords: MCDM; TOPSIS; Fuzzy numbers; Fuzzy positive ideal solution; Fuzzy negative ideal solution

1. Introduction

Multi-criteria decision making has been one of the fastest growing areas during the last decades depending on the changings in the business sector. Decision maker(s) need a decision aid to decide between the alternatives and mainly excel less preferrable alternatives fast. With the help of computers the decision making methods have found great acceptance in all areas of the decision making processes. Since multicriteria decision making (MCDM) has found acceptance in areas of operation research and management science, the discipline has created several methodologies. Especially in the last years, where computer usage has increased significantly, the application of MCDM methods has considerably become easier for the users the decision makers as the application of most of the methods are corresponded with complex mathematics. In discrete alternative multicriteria decision problems, the primary concern for the decision aid is the following:

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(1) choosing the most preferred alternative to the decision maker (DM),
(2) ranking alternatives in order of importance for selection problems, or
(3) screening alternatives for the final decision.

The general concepts of domination structures and non-dominated solutions play an important role in describing the decision problems and the decision maker's revealed preferences described above [10]. So far, various approaches have been developed as the decision aid (see, for example [9]). That is, for many such problems, the decision maker wants to solve a multiple criteria decision making (MCDM) problem. In MCDM problems, there does not necessarily exist the solution that optimizes all objectives functions, and then the concept which is called Pareto optimal solution (or efficient solution) is introduced. Usually, there exist a number of Pareto optimal solutions, which are considered as candidates of final decision making solution. It is an issue how decision makers decide one from the set of Pareto optimal solutions as the final solution (see, for more details [6]). A MCDM problem can be concisely expressed in matrix format as

	C_1	C_2	$\dots$	C_n
A_1	x_{11}	x_{12}	$\dots$	x_{1n}
A_2	x_{21}	x_{22}	$\dots$	x_{2n}
A_m	x_{m1}	x_{m2}	$\dots$	x_{mn}

$$W = [w_1, w_2, \dots, w_n],$$

where $A_1, A_2, \dots, A_m$ are possible alternatives among which decision makers have to choose, $C_1, C_2, \dots, C_n$ are criteria with which alternative performance are measured, x_{ij} is the rating of alternative A_i with respect to criterion C_j, w_j is the weight of criterion C_j.

The main steps of multiple criteria decision making are the following:

(a) establishing system evaluation criteria that relate system capabilities to goals;
(b) developing alternative systems for attaining the goals (generating alternatives);
(c) evaluating alternatives in terms of criteria (the values of the criterion functions);
(d) applying a normative multicriteria analysis method;
(e) accepting one alternative as "optimal" (preferred);
(f) if the final solution is not accepted, gather new information and go into the next iteration of multicriteria optimization.

Steps (a) and (e) are performed at the upper level, where decision makers have the central role, and the other steps are mostly engineering tasks. For step (d), a decision maker should express his/her preferences in terms of the relative importance of criteria, and one approach is to introduce criteria weights. This weights in MCDM do not have a clear economic significance, but their use provides the opportunity to model the actual aspects of decision making (the preference structure). Technique for order performance by similarity to ideal solution (TOPSIS) [7], one of known classical MCDM method, was first developed by Hwang and Yoon [4] for solving a MCDM problem. TOPSIS, known as one of the most classical MCDM methods, is based on the idea, that the chosen alternative should have the shortest distance from the positive ideal solution and on the other side the farthest distance of the negative ideal solution. The TOPSIS-method will be applied to a case study, which is described in detail. In classical MCDM methods, the ratings and the weights of the criteria are known precisely [3,4]. A survey of the methods has been presented in Hwang and Yoon [4]. In the process of TOPSIS, the performance ratings and the weights of the criteria are given as exact values. Recently, Abo-sinna and Amer [1] extend TOPSIS approach to solve multi-objective nonlinear programming problems. Jahanshahloo et al. [5] extend the concept of TOPSIS to develop a methodology for solving multi-criteria decision-making problems with interval data. In real-word situation, because of incomplete or non-obtainable information, for example, human judgements including preferences are often vague and cannot estimate his preference with an exact numerical data, the data (attributes) are often not so deterministic, there for they usually are fuzzy/imprecise [2,11], so, we try to extend TOPSIS for fuzzy data.