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A comparison study between the modified decomposition method and the traditional methods for solving nonlinear integral equations

Abdul-Majid Wazwaz

Department of Mathematics and Computer Science, Saint Xavier University, 3700 W. 103rd Street, Chicago, IL 60655, United States

Abstract

In this paper, we conduct a comparative study between the modified decomposition method and two of the traditional methods for analytic treatment of nonlinear integral and integro-differential equations. The proper implementation of the modified method can dramatically minimize the size of work if compared to existing traditional techniques. The analysis is accompanied by examples that demonstrate the comparison and show the pertinent features of the modified technique.

Keywords: Fredholm integral equations; Volterra integral equations; The modified decomposition method; The series solutions method; The direct computation method

1. Introduction

This paper presents a comparative study between the modified decomposition method and two traditional methods namely, the direct computation method and the series solution method, for solving nonlinear integral equations and nonlinear integro-differential equations. The nonlinear Fredholm integro-differential equations are given by

$$u^{(n)}(x) = f(x) + \int_0^1 K(x,t)\{R(u(t)) + N(u(t))\}\mathrm{d}t, \quad u^{(k)}(x) = b_k, \ 0 \leqslant k \leqslant (n-1), \quad n \geqslant 0 \tag{1}$$

and the nonlinear Volterra integro-differential equations are given by

$$u^{(n)}(x) = f(x) + \int_0^x K(x,t)\{R(u(t)) + N(u(t))\}\mathrm{d}t, \quad u^{(k)}(x) = b_k, \ 0 \leqslant k \leqslant (n-1), \quad n \geqslant 0, \tag{2}$$

E-mail address: wazwaz@sxu.edu

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where $u^{(n)}(x)$ is the *nth* derivative of the unknown function $u(x)$ that will be determined, $K(x,t)$ is the kernel of the integral equation, $f(x)$ is an analytic function, $R(u)$ and $N(u)$ are linear and nonlinear functions of u, respectively [1–5]. For $n = 0$, Eqs. (1) and (2) are the nonlinear Fredholm integral equations and the nonlinear Volterra integral equations, respectively.

The Volterra and Fredholm integro-differential equations (1) and (2) arise from the mathematical modeling of the spatio-temporal development of an epidemic and various physical and biological models [6] and from many scientific phenomena. Nonlinear phenomena, that appear in many applications in scientific fields, such as fluid dynamics, solid state physics, plasma physics, mathematical biology and chemical kinetics, can be modeled by PDEs and by integral equations as well.

The concepts of integral equations have motivated a huge size of research work in recent years. Several analytical and numerical methods were used such as, the direct computation method, the series solution method, the successive approximation method, the successive substitutions method and the conversion to equivalent differential equations. However, these analytical solutions methods are not easy to use and require tedious works and knowledge. The Adomian decomposition method [7,8] has been proved to be effective and reliable for handling differential, ordinary and partial, and integral equations, linear or nonlinear. It was observed by Maleknejad and Hadizadeh [9] that a complicated term $f(x)$ can cause difficult integrations and proliferation of terms in Adomian recursive scheme. In [10], the modified decomposition method was applied for a reliable treatment of the mixed Volterra–Fredholm integral equations to extend the work in [9].

In this work, we aim to conduct a comparative work between the modified decomposition method [11] and two traditional methods for analytic treatments of nonlinear integral equations and nonlinear integro-differential equations as well. Although the modified form introduces a slight change in the formulation of Adomian recursive relation [10–18], but it provides a qualitative improvement over standard Adomian method. The modified decomposition method, well-addressed in [10–18], has a constructive attraction in that it provides the exact solution by computing only very few iterations, mostly two iterations, of the decomposition series. In addition, the modified technique may give the exact solution for nonlinear equations without any need to the so-called Adomian polynomials. While this slight variation is rather simple, it does demonstrate the reliability and the power of the modified method.

In this paper, only a brief discussion of the modified decomposition method will be emphasized, because the complete details of the method are found in [10–18] and in many related works. Moreover, we will also give a brief discussion of the two traditional methods, namely the direct computation method [13] and the series method [1] that will be employed for the comparison goal.

2. The modified decomposition method

Without loss of generality, we can demonstrate the modified decomposition strategy on the nonlinear Volterra integro-differential equation (2)

$$u^{(n)}(x) = f(x) + \int_0^x K(x,t)\{R(u(x)) + N(u(x))\}\mathrm{d}t, \quad u^{(k)}(0) = b_k,\ 0 \leqslant k \leqslant (n-1), \tag{3}$$

where $u^{(n)}$ is the nth derivative of $u(x)$, and b_k are the initial conditions. Integrating both sides of Eq. (3) n times we obtain

$$u(x) = \sum_{k=0}^{n-1} \frac{1}{k!} b_k u^k + L^{-1}(f(x)) + L^{-1}\left(\int_0^x K(x,t)R(u(x)) + N(u(x))\mathrm{d}t\right), \tag{4}$$

where L is assumed invertible and L^{-1} is an n-fold integral operator.

The standard Adomian method [7,8] defines the solution $u(x)$ by the series

$$u(x) = \sum_{n=0}^{\infty} u_n(x), \tag{5}$$