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Polynomial time second order Mehrotra-type predictor–corrector algorithms

Maziar Salahi [a,*], Nezam Mahdavi-Amiri [b]

[a] Department of Mathematics, Faculty of Sciences, The University of Guilan, Namjoo Street, Rasht, Iran
[b] Department of Mathematical Sciences, Sharif University of Technology, Tehran, Iran

Abstract

Salahi et al. [M. Salahi, J. Peng, T. Terlaky, On Mehrtora type predictor–corrector algorithms, Technical Report 2005/4, Advanced Optimization Lab, Department of Computing and Software, McMaster University, http://www.cas.mcmaster.ca/~oplab/publication, SIAM Journal on Optimization, submitted for publication] give a numerical example showing that Mehrotra's original predictor–corrector algorithm, which is the basis of interior point methods software packages, may be very inefficient in practice. This motivated Salahi et al. to come up with a safeguarded algorithm that enjoys a polynomial iteration complexity and is efficient in practice. Here we discuss a variation of Mehrotra's second order predictor–corrector algorithm [S. Mehrotra, On the implementation of a (primal–dual) interior point method, SIAM Journal on Optimization 2 (1992) 575–601] and use the example of Salahi et al. to show that the algorithm may have to take very small steps in order to remain in a certain neighborhood of the central path and subsequently needs excessively large number of iterations to stop. We then introduce a safeguard that guarantees a lower bound for the maximum step size in the corrector step of the algorithm and subsequently a polynomial number of iterations. A second modification of algorithm is proposed which enjoys even a better iteration complexity. Some limited encouraging computational results are reported.

Keywords: Linear optimization; Predictor–corrector methods; Interior point methods; Second order methods; Polynomial complexity

1. Introduction

Mehrotra-type predictor–corrector algorithms are the base of the interior point methods (IPMs) software packages such as [1,11,12] and many others. Due to the practical efficiency of the algorithm the authors of [7] analyzed a feasible variant of Mehrotra's original algorithm. By a numerical example they showed that this algorithm which is using an adaptive update of the barrier (centering) parameter at each iteration might be very inefficient in practice. This observation motivated them to combine the algorithm with some safeguards to prevent such a phenomenon. In this paper we analyze a feasible version of a variation of Mehrotra's second

* Corresponding author. Address: Advanced Optimization Laboratory, McMaster University, 1280 Main Street West, Hamilton, Ont., Canada L8S4KI.
E-mail address: msalahi@optlab.mcmaster.ca (M. Salahi).

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order algorithm [3] that has been considered in [10]. Before going into the details of the algorithm, first we briefly review the basic and unique results of IPMs.

Throughout the paper we consider primal–dual IPMs to solve the following *linear optimization* (LO) problem:

$$(\mathrm{P}) \quad \min_{x\in\mathscr{R}^n}\{c^{\mathrm{T}}x : Ax = b,\ x \geqslant 0\},$$

where $A \in R^{m\times n}$ satisfies $\mathrm{rank}(A) = m$, $b \in R^m$, $c \in R^n$, and its dual problem

$$(\mathrm{D}) \quad \max_{y\in\mathscr{R}^m, s\in\mathscr{R}^n}\{b^{\mathrm{T}}y : A^{\mathrm{T}}y + s = c,\ s \geqslant 0\}.$$

Without loss of generality [5] we may assume that both (P) and (D) satisfy the interior point condition (IPC), i.e., there exists an (x^0, s^0, y^0) such that

$$Ax^0 = b, \quad x^0 > 0, \qquad A^{\mathrm{T}}y^0 + s^0 = c, \quad s^0 > 0.$$

Under IPC, finding optimal solutions of (P) and (D) is equivalent to solving the following system:

$$\begin{aligned} &Ax = b, \quad x \geqslant 0,\\ &A^{\mathrm{T}}y + s = c, \quad s \geqslant 0,\\ &Xs = 0, \end{aligned} \tag{1}$$

where $X = \mathrm{diag}(x)$. The basic idea of primal–dual IPMs is to replace the third equation in (1) by the parameterized equation $Xs = \mu e$, where e is the all one vector. This leads to the following system:

$$\begin{aligned} &Ax = b, \quad x \geqslant 0,\\ &A^{\mathrm{T}}y + s = c, \quad s \geqslant 0,\\ &Xs = \mu e. \end{aligned} \tag{2}$$

If the IPC holds, then for each $\mu > 0$, system (2) has a unique solution. This solution, denoted by $(x(\mu), y(\mu), s(\mu))$, is called the μ-center of the primal–dual pair (P) and (D). The set of μ-centers with all $\mu > 0$ gives *the central path* of (P) and (D) [2,8]. It has been shown that the limit of the central path (as μ goes to zero) exists. Because the limit point satisfies the complementarity condition, it naturally yields optimal solutions for both (P) and (D), respectively [5]. One may consult [5,9] for more details on algorithmic developments.

Now, we briefly describe the variation of Mehrotra's second order predictor–corrector algorithm which is the focus of this paper. In the predictor step, the affine scaling search direction,

$$\begin{aligned} &A\Delta x^{\mathrm{a}} = 0,\\ &A^{\mathrm{T}}\Delta y^{\mathrm{a}} + \Delta s^{\mathrm{a}} = 0,\\ &s\Delta x^{\mathrm{a}} + x\Delta s^{\mathrm{a}} = -Xs \end{aligned} \tag{3}$$

is computed and the maximum step size in this direction is calculated so that

$$(x + \alpha_{\mathrm{a}}\Delta x^{\mathrm{a}}, s + \alpha_{\mathrm{a}}\Delta s^{\mathrm{a}}) \geqslant 0.$$

However, the algorithm does not take such a step right away. It uses the information from the predictor step and the second derivative of the primal dual trajectory to compute the centering direction given by

$$\begin{aligned} &A\Delta x = 0,\\ &A^{\mathrm{T}}\Delta y + \Delta s = 0,\\ &s\Delta x + x\Delta s = \mu e - \Delta x^{\mathrm{a}}\Delta s^{\mathrm{a}}, \end{aligned} \tag{4}$$

where μ is defined adaptively as

$$\mu = \left(\frac{g_{\mathrm{a}}}{g}\right)^2 \frac{g_{\mathrm{a}}}{n},$$