Asymptotic analysis of channel flows with slip lengths that depend on the pressure

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Abstract

An asymptotic analysis of (two-dimensional) channel flows of incompressible fluids with a slip length that depends on the pressure and/or the axial pressure gradient is performed, and analytical solutions of the leading-order equations for several slip lengths are presented. It is shown that, in general, the pressure drop is not a linear function of the distance along the channel but depends in a nonlinear fashion on both the inlet and outlet pressures. It is also shown that, when the slip length depends on both the pressure and the axial pressure gradient, the pressure distribution can only be determined numerically. A thermal analysis of hydrodynamically fully developed channel flows with slip lengths that depend on the pressure is also reported, and the slip model is generalized to the case that the dynamic viscosity of the fluid is a function of the local pressure in Appendix.
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1. Introduction

During the past 20 years or so, there has been a revival of rarefied gas dynamics as related to flows in micro-channels and their applications in micro-electro-mechanical systems (MEMS) [1] where the dimensions of the devices, e.g., channels, are comparable to the mean free path of the gas flowing through them, microfluidics-based biomedical separation and diagnostic techniques. According to the value of the Knudsen (Kn) number (defined as the ratio of the gas mean free path to the channel height), gas flows can be classified as continuum ($Kn < 0.01$), slip ($0.01 < Kn < 0.1$), transition ($0.1 < Kn < 10$) and free-molecular ($Kn > 10$). Only in the continuum gas regime are the well-known conservation equations of mass, linear momentum and energy applicable, although many authors have used these equations to analyze flows in the slip and transition regimes, whereas the free-molecular regime has to be dealt with methods based on molecular dynamics or the Boltzmann equation.

In the slip regime, Maxwell proposed in 1879 to use the well-known continuum conservation equations but with a simple formula to determine the slip velocity. Such a simple formula defines a slip length that depends on the mean free path and the accommodation of the molecules on the solid walls depending on whether the molecules reflect from these surfaces on a specular (angle of incidence equal to angle of reflection) or diffuse manner. A few years later, it was also found that the temperature in the slip regime also undergoes a jump at solid walls which also depends on the mean free path. Still later, thermal creep effects, i.e., slip flows caused by temperature gradients along the walls, were also reported for gases.

For ideal gases, the mean free path is $\frac{RT}{\sqrt{2}\pi d^2 N_A p}$ where R is the universal gas constant, T and p are the gas temperature and pressure, respectively, d is the molecular diameter, and N_A is the Avogrado number. Therefore, the molecular free path increases inversely proportional to the pressure. Moreover, if one were to use Maxwell's slip model or generalizations of it [2], one would find that the slip length increases as the pressure decreases and, eventually, the mean free path might become much larger than the channel height so that the slip flow for which this simple slip formula was developed is no longer applicable.

Although the concept of slip and several formulae for it have been developed for gases in the literature with the help of insight, detailed molecular dynamics calculations and fitting of experimental data [1,2], similar concepts have been introduced to analyze flows of liquids in microchannels despite the fact the concept of mean free path in a liquid is not well-defined, and some of these flows have been considered also under non-isothermal conditions considering or disregarding temperature jump boundary conditions analogous to those developed for gases.

The no-slip boundary condition at a solid–liquid interface has been of paramount importance for our understanding of fluid dynamics. This condition, however, is an assumption that cannot be derived from first principles and could be violated [3–5]. One, in fact, should expect a complex behavior at a liquid–solid interface, involving an interplay of many physico-chemical parameters, including wetting, shear rate, pressure, surface charge, surface roughness, impurities, dissolved gases, surface chemistry, fluid–surface intermolecular interactions, etc. [5–7]. In fact, it has been shown that local intermolecular interactions dominate when the solid surface is smooth, but roughness dominates otherwise, and the critical shear stress and shear rate have been observed to deviate from predictions based on the use of the no-slip boundary condition in an exponential manner with increasing surface roughness [8]. In addition, experiments have shown that physisorbed surfactants can change the hydrodynamic boundary condition of oil flow from stick, i.e., no-slip, to partial slip, provided that the shear stress on the wall exceeds a threshold value that decreases with increasing surface coverage of the surfactant.

In flows of water in microchannels, it has been observed that, if the surface is hydrophilic (e.g., uncoated gas), the measured velocity profiles are consistent with the imposition of the no-slip condition at the wall [9]. However, when the microchannel surface is covered with a monolayer of a hydrophobic substance, an apparent velocity slip has been observed just above the solid surface. Walther et al. [10] used non-equilibrium molecular dynamics simulations to analyze plane Couette flows of water with hydrophobic (graphite) surfaces and showed that the slip length decreases with increasing the system pressure. They also showed that changing the properties of the interface from hydrophobic to strongly hydrophilic reduces the slip length. Slip has also been observed in inelastic liquids, specially engineered surfaces, entangled polymers, etc. In the latter, there is evidence to support both adhesive failure at the melt/metal interface and cohesive failure within the entangled polymer melt.

The results of Walther et al. [10] and others remind us of the static and dynamic friction forces between two solids in contact. When two solid bodies are in (dry) contact, the friction or tangential force opposes the motion or the tendency to move. This tendency is due to the micro-irregularities, i.e., the irregular ridges and valleys, of the two surfaces and the opposing force to motion at this level is not necessarily along the mean contact surface. The net friction force is equal to the applied tangential force as long as it is less than a limiting force and, hence, the solid body does not move. If the applied tangential force is greater than this threshold value, the body will start moving, but then the frictional force known as the dynamic friction is slightly less than the maximum static friction because the contact locations at the microlevel are continuously changing. It is well-known that polishing a solid surface leads to a decrease in the dry friction.

The present paper differs from previous treatments of incompressible flows in microchannels in that no use is made whatsoever of the mean free path which as stated previously is not well-defined for liquids. Instead,