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$BMAP/G/1/N$ queue with vacations and limited service discipline

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Abstract

We consider a finite buffer single server queue with batch Markovian arrival process ($BMAP$), where server serves a limited number of customer before going for vacation(s). Single as well as multiple vacation policies are analyzed along with two possible rejection strategies: partial batch rejection and total batch rejection. We obtain queue length distributions at various epochs and some important performance measures. The Laplace–Stieltjes transforms of the actual waiting time of the first customer and an arbitrary customer in an accepted batch have also been obtained.

Keywords: Finite buffer; Queue; Batch Markovian arrival process; Vacation; Limited service discipline

1. Introduction

The areas of communications, manufacturing, and transportation all give rise to queueing systems involving complex, nonrenewal arrival processes. Traditional teletraffic analysis using Poisson process is not powerful enough to capture the correlated and bursty nature of traffic arising in the present high-speed networks, e.g., in ATM networks packets or cells of voice, video and data are sent over a common transmission channel on statistical multiplexing basis. The performance analysis of statistical multiplexers whose input consists of a superposition of several packetized sources have been done through some analytically tractable arrival process, viz., batch Markovian arrival process ($BMAP$) introduced by Lucantoni [8]. This type of arrival process includes many familiar input processes such as Markovian arrival process (MAP) [9], Markov modulated Poisson process ($MMPP$), PH-type renewal process, Interrupted Poisson process (IPP), Poisson Process etc.

Queueing systems with vacations are considered to be an effective instrument in modelling and analysis of communication networks and several other engineering systems in which single server is entitled to serve more than one queue. Modelling such systems as single server queues with vacations allows one to analyze each workstation in relative isolation since the time the server is attending to other stations in the system may be modeled

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as vacation. More detail on this topic can be found in the comprehensive survey by Doshi [3]. Vacation models are distinguished by their scheduling disciplines, that is, the rules determining when a service stops and a vacation begins, e.g., exhaustive, limited, gated etc. An extensive amount of literature is available on infinite and finite buffer $M/G/1$ type vacation models and can be found in Takagi [13,14], respectively.

In recent years there has been a great interest in analyzing queueing systems with server vacations and $BMAP$ as input process: $BMAP/G/1$ queue, e.g., Lucantoni [8], Matendo [10], Ferrandiz [4] etc. As queueing analysis of finite systems are more realistic in applications than infinite systems, the detail study of $BMAP/G/1/N$ queue with vacations under exhaustive service discipline was performed by Niu et al. [12], where they have included setup time, close-down time, single/multiple vacations.

In this paper we consider a $BMAP/G/1/N$ queue with single (multiple) vacation(s) under limited service discipline i.e., a fixed limit L is placed before the server and the server will at most serve L customers during a busy period before going for vacation(s). The service discipline discussed is more or less similar to that of the timed token protocol of FDDI and token passing ring of LAN, in which vacation would correspond to the time the token is away at other stations. Batches which upon arrival find not enough space in the buffer are, either fully rejected, or a part of that batch is rejected. Some queueing protocol are based on the former strategy and it is known as whole batch acceptance strategy (WBAS) or total rejection policy. Later one is known as the partial batch acceptance strategy (PBAS) or partial rejection policy. Both the policies are analyzed in this paper along with queue length distributions at various epochs and the LST's of actual waiting time of the first customer and an arbitrary customer in an accepted batch. The analysis have been carried out in an unified way taking into consideration of single/multiple vacation policy, partial and total rejection policy by defining two indicator functions, δ_S and δ_T given below.

$$\delta_S = \begin{cases} 1, & \text{for single vacation policy,} \\ 0, & \text{for multiple vacation policy,} \end{cases} \qquad \delta_T = \begin{cases} 1, & \text{for total rejection policy,} \\ 0, & \text{for partial rejection policy.} \end{cases}$$

By fixing $\delta_S = 1$ and $\delta_T = 0$, we obtain the results for partially rejected $BMAP/G/1/N$ queue with single vacation policy and similarly, the results for the rest of the strategies can easily be obtained.

So far, very few literature is available on the infinite or finite capacity $MAP/G/1$ queue with vacations under limited service discipline. The main reason for this is that the problem itself is combinatorially very complex, and complexity further increases when batch Markovian arrival process and finite buffer space are considered. In this direction Blondia [2] carried out the analysis of $MAP/G/1/N$ queue with multiple vacations and limited service discipline. Recently Gupta et al. [5] considered the same queue with multiple as well as single vacation under limited service discipline. A more general study on limited service discipline has been carried out by Banik et al. [1]. They considered $MAP/G/1/N$ queue with single and multiple vacation(s) under E-limited with limit variation service (ELV), where the server serves until either the system emptied or a randomly chosen limit of $l(0 \leqslant l \leqslant L)$ customers have been served. As a final remark it may be mentioned that although the mathematical approach adopted in this paper is somewhat similar to our previous papers [5,1] but the extension process is not straightforward. There exist a lot of special issues in deriving mathematical formulae and explaining their physical meanings for the $BMAP$ case which made the model more complex and forced significant extension both in terms of theoretical and computational aspects. The purpose of this paper is to treat the problem as a general queueing problem and obtain explicit mathematical results which can be easily computable by the practitioners/engineers so that they can make use of those results.

The paper is organized as follows. In Section 2 we give the description of the model and introduce the notations used to describe the model parameters. In Section 3, we present the analytic analysis of the model and obtain queue length distributions at various epochs. In Section 4 and 5, we derive performance measures and describe the computational procedure along with numerical results, respectively. Numerical result have been presented to study the behaviour of some performance measures against the variation of critical model parameters.

2. Description of the model

Let us consider a $BMAP/G/1/N$ queue where N is the capacity of the queue. The server is allowed to serve a maximum of L customers during each visit to the queue, i.e., the server goes for a vacation if either the queue