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The method of fundamental solutions for solving options pricing models

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Abstract

This paper provides a foundation of the method of fundamental solutions (MFS) for the Options Pricing models governed by the Black–Scholes equation in which both the European option and American options are considered. In the solution procedure, no artificial boundary conditions are imposed for both datum and infinite sides of the stock prices. In the cases of the European options, no time marching procedures are required and numerical results are verified with the exact solutions. Since the free boundary conditions are considered for the American options, boundary update procedure is thus applied. At the same time, numerical results are compared with the results in the literatures. These numerical results indicate the MFS is an effective and robust meshless numerical solution for solving the Options Pricing models.

Keywords: Black–Scholes equations; Method of fundamental solutions; American options; Artificial boundary conditions; Free boundary conditions

1. Introduction

We consider the Black–Scholes model for both European and American options, which is firstly proposed in 1973 by Black and Scholes [1]. It is well known that American options involve the so-called free boundary conditions. Accordingly, closed form solutions rarely exist and numerical methods should be applied. Traditional numerical methods have been applied to the Black–Scholes model successfully. Geske and Shastri [2] as well as Wu and Kwok [3] developed the finite difference method for the American options valuation. On the other hand, Cox et al. [4] utilized the binomial method. In addition, Huang et al. [5] adopted the integral equation method. Moreover, Broadie and Detemple [6] wrote a review paper in the comparison for some of these numerical methods.

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Recently, Hon and Mao [7] as well as Marcozzi et al. [8] utilized the multi-quadric method (MQ) to solve the Black–Scholes model successfully. Roughly speaking, meshless numerical methods can be divided into two categories. The first one is domain-type method in which both the differential equations and boundary conditions are approximated, such as the MQ, and the second one is boundary-type method where only boundary conditions are collocated. In this paper, we develop the method of fundamental solutions (MFS), which is a boundary-type meshless numerical method, to solve the Black–Scholes model.

The MFS, which is first proposed in 1964 by Kupradze and Aleksidze [9], has become a versatile numerical scheme to solve well-posed problems. This method is free from treatments of singularities, meshes, and numerical integrations. The concept of the MFS is to decompose the solutions of the partial differential equations by superposition of the fundamental solutions with proper intensities. Here, the intensities are the unknown parameters, which can be obtained by collocating known augmented data on the boundary. In 1995, Golberg [10] used the MFS to obtain numerical solutions of the Poisson equation. Fairweather and Karageorghis [11] and Fairweather et al. [12] solved the elliptic boundary value problems and scattering and radiation problems, respectively, by the MFS. Overall, Fairweather and Karageorghis [11], Fairweather et al. [12] as well as Tsai [13] have written some review articles for the MFS.

The Black–Scholes model is a partial differential equation, which can be transformed to the advection–diffusion equation as addressed by Marcozzi et al. [8]. Chen et al. [14] applied the MFS for diffusion equations by using the modified Helmholtz fundamental solution. On the other hand, Tsai [13] and Young et al. [15,16] solved the diffusion equation by the MFS based on the diffusion fundamental solution. In this paper, we provide a general formulation of the MFS based on the advection–diffusion fundamental solution to solve the Options Pricing modes of multi-assets. In our formulations, the far field boundary conditions are automatically satisfied by the fundamental solution. Moreover, Marcozzi et al. [8] proved no artificial datum boundary condition should be applied. Thus, no time marching procedures are required in our formulation of the European options. For the American options, boundary update procedure is exploited to solve the free boundary conditions.

The organization of this paper is as follows. In Section 2, we consider the MFS formulations for European options of multi-assets. In Section 3, the formulation is extended to American options. Numerical results and discussions are addressed in Section 4. In this section, the following numerical experiments are carried out: European options of single asset, European options of two assets, and American options of single asset. Then, conclusions are stated in Section 5.

2. MFS formulations for European options

2.1. Canonical form

This section provides the MFS formulation for European Options Pricing model. A bracket option is an option whose price is based on multiple underlying assets. We assume there are n such assets whose price at time t is denoted by $S_1(t), S_2(t), \ldots, S_n(t)$. The value $P(S_1, S_2, \ldots, S_n, t)$ of the European option can be determined by solving the following partial differential equation:

$$\frac{\partial P}{\partial t} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{ij}\sigma_i\sigma_j S_i S_j \frac{\partial^2 P}{\partial S_i \partial S_j} + \sum_{i=1}^{n} r S_i \frac{\partial P}{\partial S_i} - rP = 0, \tag{1}$$

where σ_i^2 is the volatility of assets i, ρ_{ij} is the correlation between asset i and j, and r is the risk free interest rate. Let T be the time of expiry and X be the exercise price of the option, we then have the following terminal condition:

$$P(S_1, S_2, \ldots, S_n, T) = F(S_1, S_2, \ldots, S_n, X), \tag{2}$$

where $F(S_1, S_2, \ldots, S_n, X)$ is the payoff function. In the numerical experiments of this paper, we adopt $F(S, X) = \max\{X - S, 0\}$ for one asset as proposed by Wilmott [17] and $F(S_1, S_2, X) = \max\{X - \min\{S_1, S_2\}, 0\}$ for two assets, whose exact solutions can be found in Stultz [18] and Johnson [19]. In which, the former is defined by *MODEL I* and the later is denoted by *MODEL II*.