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# Solve partial differential equations by two or more radial basis functions

Yunxin Zhang

*School of Mathematics, Fudan University, Shanghai 200433, China*

**Abstract**

There are many partial differential equations, their solution might be oscillatory or vary rapidly. We will get an inaccurate result or encounter an ill-conditioned problem whenever we solve it using Kansa's collocation method, MFS or other methods. In this paper, we will present a new method, in which such PDEs will be solved by two or more radial basis functions (RBFs). So these PDEs can be solved efficiently and accurately.


*Keywords:* RBF; PDEs; Meshless

## 1. Introduction

In the last decade, there has been some advance developments in applying the radial basis functions (RBFs) for the numerical solutions of various types of partial differential equations (PDEs). The initial development was due to the pioneering work of Kansa [1] who directly collocated the RBFs for the approximated solutions of the equations. In general, the Kansa's method has several advantage over the widely used FEM, in that

(1) it is a truly meshless in which the collocation points can be choose freely (no connectivity between points is required as FEM). Hence the complicated meshing problem has been avoid;
(2) it is spatial dimension independent which can easily be extended to solve high dimensional problems.

Despite the many special attractive features of RBFs, it is known that most of the RBFs are globally defined basis functions. This means that the resulting matrix for interpolation is dense and can be highly ill-conditioned, especially for a large number of interpolation points in 3D. This poses serious stability problems and high computational cost. At the same time, the CS-RBFs also have several difficulties: (i) the accuracy and efficiency depends on the scale of the support and determining the scale of support is uncertain; (ii) the convergence rate of CS-RBFs is low. In order to obtain a sparse matrix system, the support needs to be

*E-mail address:* yunxin_z@21cn.com

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small; then the interpolation error become unacceptable. When the support is large enough to make the error acceptable, the matrix system becomes dense and the advantages to the traditional RBFs are lost.

In the society of meshfree RBF method, there are many techniques to circumvent this problems. To my knowledge, the most important techniques might be DDM [2–6], precondition method [7], MLS [8–11], you can also find some good method in [12–16]. In the literature of FEM, some useful techniques also can be find in [28–30].

There are many partial differential equations, their solution might be oscillatory or vary rapidly. In this paper, we will present a new numerical method, in which different RBFs are used in different subdomains. Combined with BKM–DRM, we then get an efficient and accurate numerical method. The organization of this paper is as follows: In Section 2 we introduce some elementary knowledge about BKM, the method of interpolation by different RBFs in different subdomains is presented in Section 3, the numerical results are listed in Section 4, and conclusions in Section 5.

## 2. Basic knowledge about BKM

In this section, we briefly introduce some elementary knowledge about BKM, for more details see [18–23] and references therein.

Like the DRBEM and MFS, the BKM can be viewed as a two-step numerical scheme, namely, DRM and RBF approximation to particular solution and the evaluation of homogeneous solution. The latter is gotten by BKM. For the sake of completeness, here we outline the basic methodology to approximate a particular solution. Let us consider the differential equation

$$L\{u(x)\} = f, \quad x \in \Omega \tag{1}$$

with boundary conditions

$$u(x) = b_1(x), \quad x \in \Gamma_u, \tag{2}$$

$$\frac{\partial u(x)}{\partial n} = b_2(x), \quad x \in \Gamma_T, \tag{3}$$

where $L$ is a differential operator, $f(x)$ is a known forcing function, and $n$ is the unit outward normal. $x \in R^d$, $d$ is the dimension of geometry domain, which is bounded by a piecewise smooth boundary $\Gamma = \Gamma_u + \Gamma_T$. In order to facilitate discussion, it is assumed here that the operator includes the Laplace operator, namely,

$$L\{u\} = \nabla^2 u + L_1\{u\} \tag{4}$$

from [17] we see that this assumption is not necessary, Eq. (1) can be restated as

$$\nabla^2 u + u = f(x) + u - L_1\{u\}. \tag{5}$$

The solution of the above Eq. (5) can be expressed as

$$u = v + u_p, \tag{6}$$

where $v$ and $u_p$ are the general and particular solutions, respectively. The latter satisfies the equation

$$\nabla^2 u_p + u_p = f(x) + u_p - L_1\{u_p\} \tag{7}$$

but does not necessarily satisfy boundary conditions (2) and (3). $v$ is the homogeneous solution of the Helmholtz equation

$$\nabla^2 v + v = 0, \quad x \in \Omega, \tag{8}$$

$$v(x) = b_1(x) - u_p(x), \quad x \in \Gamma_u, \tag{9}$$

$$\frac{\partial v(x)}{\partial n} = b_2(x) - \frac{\partial u_p(x)}{\partial n}, \quad x \in \Gamma_T. \tag{10}$$

The first step in the BKM is to evaluate the particular solution up by the DRM and RBF. After this, Eqs. (8)–(10) can be solved by the boundary RBF methodology using the nonsingular general solution.