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Existence and global exponential stability of periodic solution for discrete-time BAM neural networks ☆

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Abstract

The discrete-time analogues of bidirectional associative memory neural networks with periodic coefficients and distributed delays are formulated and studied. And by using the continuation theorem of coincidence degree theory, we derive the existence of periodic solution for the discrete-time BAM neural networks. And by constructing a appropriate Lyapunov-type sequence, we prove the global exponential stability of the periodic solution for the model. It is shown that the discrete-time analogues inherit the existence and global exponential stability of periodic solution for the continuous-time BAM neural networks. An example is given to illustrate the effectiveness of the obtained results.

Keywords: BAM neural networks; Periodic solution; Global exponential stability; Discrete-time analogues

1. Introduction

The bidirectional associative memory neural networks (BAM), which were first introduced by Kosto [1,2], have attracted great attention of many researchers. The BAM neural networks have been found useful in areas of image processing, pattern recognition, and automatic control [3–5]. Because the addressable memories or patterns are stored as stable equilibria and stable periodic solutions, it is important to investigate the equilibria and periodic solutions of the BAM neural network. Thus Refs. [6–10] studied the existence and stability of the equilibrium of the continuous-time BAM neural networks, and Refs. [11–15] studied the existence and stability of the periodic solution for the networks. For example, Ref. [15] investigated the existence and global exponential stability of periodic solution for BAM neural networks with periodic coefficients and continuously distributed delays modelled by the following system:

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$$\frac{\mathrm{d}x_{ki}}{\mathrm{d}t} = -a_{ki}(t)x_{ki}(t) + \sum_{j=1}^{p(3-k)} w_{(3-k)ji}(t)f_{(3-k)j}\left(\int_0^{+\infty} g_{(3-k)j}(s)x_{(3-k)j}(t-s)\mathrm{d}s\right) + I_{ki}(t). \tag{1}$$

where $p(k) = \begin{cases} m_1, & k=1, \\ m_2, & k=2. \end{cases}$ And $k=1$ represent domain A, $k=2$ represent domain B; $a_{ki}(t)>0$, $x_{ki}(t)$ represent the activation of the ith neuron in the domain k at the time t, f_{kj} are signal transmission functions of the jth neuron in the domain k, $w_{kji}(t)$ are the connection weights of the jth neuron in the domain k to the ith neuron in the domain $3-k$, $I_{ki}(t)$ correspond to the external inputs of the ith neuron in the domain k. And $a_{ki}(t)$, $w_{kji}(t)$, $I_{ki}(t)$ are T-periodic functions in $\mathbb{R}$.

In fact, discrete-time neural networks are more important than their continuous-time counterparts in implementing and applications of neural networks. For example, Lee and Kanan both employed a discrete-time BAM neural network in their research work [3,4]. In addition, it is essential to formulate discrete-time counterparts of the continuous-time networks when one wants to simulate or compute the continuous-time system after he has obtained its dynamical characteristics. We may refer to Refs. [17–23] for more details about the investigating importance of discrete-time neural networks.

Refs. [18–20,23] considered the discrete-time BAM neural networks. Ref. [18] studied the existence and exponential stability of a unique equilibria of continuous-time BAM neural networks with constant delays and their discrete-time analogues. Ref. [23] studied the existence, uniqueness and global exponential stability of the equilibrium point of discrete-time BAM neural networks with variable delays. And Ref. [19] studied discrete-time analogues of BAM neural networks described by integro-differential equations modellings [6]

$$\begin{aligned} \frac{\mathrm{d}x_i}{\mathrm{d}t} &= -a_i x_i(t) + \sum_{j=1}^{m} b_{ij} S\left(\int_0^{+\infty} K_{ij}(s) y_j(t-s)\mathrm{d}s\right) + I_i, \\ \frac{\mathrm{d}y_i}{\mathrm{d}t} &= -c_i y_i(t) + \sum_{j=1}^{m} d_{ij} S\left(\int_0^{+\infty} H_{ij}(s) x_j(t-s\,\mathrm{d}s)\right) + J_i. \end{aligned} \tag{2}$$

The authors of [19] discretized the system (2) into

$$\begin{aligned} x_i(n+1) &= \mathrm{e}^{-a_i h}x_i(n) + \frac{1-\mathrm{e}^{-a_i h}}{a_i}\sum_{j=1}^{m} b_{ij} S\left(\sum_{p=1}^{+\infty} \mathscr{K}_{ij}(p) y_j(n-p)\right) + \frac{1-\mathrm{e}^{-a_i h}}{a_i} I_i, \\ y_i(n+1) &= \mathrm{e}^{-c_i h}y_i(n) + \frac{1-\mathrm{e}^{-c_i h}}{c_i}\sum_{j=1}^{m} d_{ij} S\left(\sum_{p=1}^{+\infty} \mathscr{H}_{ij}(p) x_j(n-p)\right) + \frac{1-\mathrm{e}^{-c_i h}}{c_i} J_i. \end{aligned} \tag{3}$$

By constructing discrete-time versions of Halanay-type inequalities, they obtained a set of sufficient conditions for the global exponential stability of the unique equilibrium of the discrete-time analogues (3).

But up to now, to the best of our knowledge, few studies have considered periodic solution for the discrete-time BAM neural networks. Because of the importance of the periodic solution for BAM neural networks, we shall investigate the existence and global exponential stability of periodic solution for the discrete-time analogues of the BAM neural networks (1) in this paper.

2. Discrete-time formulation

In this section we employ a semi-discretization technique in formulating discrete-time analogues of the continuous-time networks (1). This method has been widely applied to formulate discrete-time analogues of continuous-time dynamical systems [17–22]. We first approximate the integral term of system (1) with discrete sums of the following form:

$$\int_0^{+\infty} g_{(3-k)j}(s)x_{(3-k)j}(t-s)\,\mathrm{d}s \approx \sum_{\left[\frac{s}{h}\right]=1}^{+\infty} g_{(3-k)j}\left(\left[\frac{s}{h}\right]h\right)x_{(3-k)j}\left(\left[\frac{t}{h}\right]h - \left[\frac{s}{h}\right]h\right)\varphi(h)$$

for $t\in[nh,(n+1)h]$, $s\in[lh,(l+1)h]$, $n\in\mathbb{Z}_0^+ = \{0,1,2,\ldots\}$, $l\in\mathbb{Z}^+ = \{1,2,\ldots\}$, where h is a fixed positive real number denoting a uniform discretization step size, $\varphi(h)>0$ for $h>0$ and $\varphi(h) = h + O(h^2)$, $[u]$ denotes