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Different approaches for state filtering in nonlinear systems with uncertain observations

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Abstract

This paper addresses the least squares filtering problem for nonlinear systems with uncertain observations, when random interruptions in the observation process are modelled by a sequence of independent Bernoulli random variables. We propose four filtering algorithms which are based on different approximations of the first and second-order statistics of a nonlinear transformation of random vectors perturbed by additive and multiplicative noises. These algorithms generalize the extended and unscented Kalman filters to the case in which there is a positive probability that the observation in each time consists of noise alone and, hence, does not contain the signal to be estimated. The accuracy of the different approximations is also analyzed and the performance of the proposed algorithms is compared in a numerical simulation example.

Keywords: Uncertain observations; Nonlinear systems; Least squares estimation; Unscented Kalman filter; Extended Kalman filter

1. Introduction

There are many practical situations in which the signal appears in the observation in a random manner, for instance, systems where there are intermittent failures in the observation mechanism, fading phenomena in propagation channels, target tracking, accidental loss of some measurements, or inaccessibility of the data at certain times. In these situations, at each time, there is a positive probability (called *false alarm probability*) that only noise is observed and, hence, that the observation does not contain the transmitted signal. To describe these observations (called *uncertain observations*), the observation equation, with the usual additive measurement noise, is formulated by multiplying, at any sample time, the signal function by a binary random variable taking the values one and zero (Bernoulli random variable); the value one indicates that the signal is present in the observation whereas the value zero reflects the fact that the observation is only noise. Thus, the observation equation involves both an additive and a multiplicative noise which models the uncertainty about the signal being present in each observation (this is called the *uncertainty* of the observation).

Discrete-time linear systems with uncertain observations have been widely used in estimation problems related to the above practical situations, (which often appear, for example, in Communication Theory).

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Systems with uncertain observations are always non-Gaussian (even if the noises are Gaussian) and hence, as occurs in other kinds of non-Gaussian linear systems, the least squares estimator (conditional expectation of the signal given the observations) is not easily obtainable by a recursive algorithm; research in these systems, then, is focused on the search of suboptimal estimators for the signal. Much has been published on this subject and suboptimal estimators, which present significant computational advantages, have been found by some authors. The problem has been addressed under different hypotheses about the processes involved and different approaches have been used; for example, assuming that the state-space model of the signal is known, linear and polynomial estimation algorithms from uncertain observations have been obtained in [1–3], among others. On the other hand, in some situations, the state-space model of the signal is not available and another type of information must be used to address the signal estimation problem. In recent years, assuming that such a model is not complete, linear and polynomial estimation algorithms from uncertain observations have been derived using the covariance functions of the processes involved in the observation equation (see, for example [4–7] and their references).

As occurs in non-Gaussian linear systems, in nonlinear systems, the computation of the least squares estimator is extremely difficult. The development, using different approaches, of suboptimal filtering algorithms for nonlinear systems has attracted a great deal of interest in the scientific community because of their wide applicability in several practical situations such as navigational and guidance systems, radar tracking and satellite and aeroplane orbit determination [8,9]. A common way to approach the estimation problem in nonlinear systems is to linearize the systems and afterwards to apply a known filtering algorithm; in this context, the most widely used algorithms are the extended Kalman filter and modifications to the latter [10,11]. When the state-space model is not available, the filter proposed in [12] can be employed. A polynomial version of the extended Kalman filter, based on the Calerman approximation of the nonlinear system, was recently derived in [13].

Although the extended Kalman filter has been successfully applied to numerous nonlinear systems, it can present some important drawbacks involving, on the one hand, the evaluation of the Jacobian matrices and, on the other, its instability; nevertheless, if the state and measurement models present important nonlinearities, the extended Kalman filter performance can be substantially improved by using, for example, Gaussian sum filters [14–16].

Avoiding the limitations of the extended Kalman filter, another estimation technique in nonlinear systems, known as the unscented Kalman filter [17], has been shown to be a superior alternative in a variety of applications domains, including state estimation, dual estimation and parameter estimation. The unscented filter was first proposed by Julier; further developments can be seen in [18]. Unlike the extended Kalman filter, the unscented filter does not approximate the nonlinear state and observation models, but uses true nonlinear models and, by means of a minimal set of deterministically chosen sample points, captures the mean and covariance estimates; when the points are propagated through the true nonlinear system, they capture the posterior mean and covariance accurately to the second order for any nonlinearity; this approximation is obtained without the need to compute any Jacobian matrices and requires only very simple matrix operations.

The estimation problem in nonlinear systems with uncertain observations has not received much attention; in [19] the problem is approached from a covariance assignment viewpoint. Therefore, the aim in the present paper is to obtain, using different techniques, suboptimal filtering algorithms for nonlinear systems with uncertain observations; concretely, the goal is to generalize the extended and unscented Kalman filters to a nonlinear system with uncertain observations in which the Bernoulli random variables describing the uncertainty are independent.

The paper is organized as follows: the next section presents the state process, the measurement models to be considered and the formulation of the estimation problem to be treated; then, in Section 3, we describe different procedures to approximate the statistics of nonlinear transformations of random vectors perturbed by additive and multiplicative noises. Using these approximations, in Section 4 we derive different filtering algorithms for state estimation. Section 5 gives a numerical simulation example in which the performance of the proposed algorithms is compared. Finally, in two appendices, we present an overview of the Taylor series expansion of a multidimensional nonlinear function and a brief summary of the scaled unscented transform.