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Using FEM–RVT technique for solving a randomly excited ordinary differential equation with a random operator

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Abstract

The technique of stochastic finite element (SFEM) which is the finite element technique FEM adapted to stochastic problems can be re-described to use random variable transformation technique RVT. A new FEM–RVT technique was successfully used in solving stochastic problems with random excitation [M. El-Tawil, W. El-Tahhan, A. Hussein, A proposed technique of SFEM on solving ordinary random differential equation, J. Appl. Math. Comput. 161 (2005) 35–47].

In this paper, the technique is adapted to solve a randomly excited differential equation with a random operator. The technique shows high accuracy when solving a case study compared with the exact solution. Finally a problem with unknown exact solution is solved using this technique.

Keywords: FEM; Computational methods for stochastic equations; Ordinary differential equations; Stochastic methods

1. Introduction

Linear differential equations with random coefficients are important models of a lot of problems in engineering and applied sciences. These coefficients represent the properties of the system under consideration. They can be thought of as random variables or, random processes with a specified probability structure. Mathematically, the problem can be formulated as

$$\Lambda(x;\theta)[w(x;\theta)] = g(x;\theta), \tag{1}$$

where $\Lambda(x;\theta)$ is a random operator, $w(x;\theta)$ is the solution process and $g(x;\theta)$ is the random excitation function in which θ is a random outcome of a triple probability space (Θ, κ, P), where Θ is a sample space, κ is σ-algebra associated with Θ and P is a probability measure.

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The determination of the solution process $w(x;\theta)$ of the stochastic differential Eq. (1) through its probability density function (p.d.f.) or some important statistical moments, the average and the standard deviation, may be possible through using many techniques, for example; the use of Fokker–Planck equation [2], the transformation technique [3], Wiener–Hermite expansion [4,5], perturbation methods [6,7], stochastic linearization [8], WHEP technique [9], decomposition method [10], stochastic finite element [1,11–13] and others [14–16]. The application on stochastic beams was achieved by many authors, for example; [1,17–19].

In this paper, a general FEM–RVT technique (Section 2) is introduced to solve Eq. (1) and this algorithm is realized by solving a beam with random properties (bending rigidity EI) and subjected to sine load with randomness in the amplitude of the load. The technique gives high accuracy compared with the exact solutions. Finally a problem with unknown exact solution is solved using this technique.

2. FEM–RVT technique

FEM–RVT technique is a combination of the deterministic finite element method (FEM) [20] and the random variable transformation (RVT) theory [21]. In this technique, the differential equation is solved firstly using the deterministic theory of finite elements, which yields exact solutions at the nodes and interpolated solution in between. These solutions are functions of the system input random variables. Using the random variable transformation theory (RVT), we get a very good approximation for the p.d.f. of the solution process. The advantage of this technique is getting the solution p.d.f. which is not available in most of other numerical techniques. The limitations of the technique arise from the bounds of the use of random variable transformation theory and the difficulties of the computations in both the finite element and random variable transformation theories.

2.1. Brief mathematical description of the technique

The general form of a linear stochastic differential equation is

$$\Lambda_a(x;\theta)[w(x;\theta)] = g(x;\theta), \quad x \in D, \ \theta \in \Omega, \tag{2}$$

where $\Lambda_a(x;\theta)$ is a linear differential operator, with respect to space, whose coefficients $\{a_k(x;\theta)\}_{k=1}^{n-1}$ can be modeled as random processes with known closed form probability density functions, $w(x;\theta)$ is the solution process and $g(x;\theta)$ is the random excitation function. The processes $\{a_k(x;\theta)\}_{k=1}^{n-1}$ and $g(x;\theta)$ have a well known closed form joint probability density function. Also, these processes are continuous with respect to the space variable x and the forms of the corresponding functional dependence take the form:

$$g(x;\theta) = g_0(\theta)R(x), \quad \text{and} \quad a_k(x;\theta) = \beta_k(\theta)h(x). \tag{3}$$

The spatial domain of Λ_a is denoted by D, x refers to a point in this domain, and θ is a random outcome of a triple probability space (Ω, κ, P). The aim then is to solve for the response process $w(x;\theta)$ as a function of both arguments.

Using our proposed technique the following steps are performed:

(1) Since the random processes $\{a_k(x;\theta)\}_{k=1}^{n-1}$ and $g(x;\theta)$ have known explicit forms dependent on x, we can apply the deterministic finite element to get finally the FEM solution of Eq. (2) over the domain D in the form:

$$w(x;\theta) = S\left(x; g_0(\theta), \{\beta_k(\theta)\}_{k=1}^{n-1}\right), \tag{4}$$

where S is – in general – a nonlinear functional of its arguments.

(2) Eq. (4) can be considered as a transformation between the input variables $g_0, \beta_1, \beta_2, \beta_3, \ldots, \beta_{n-1}$ to the transformation and the output process (the solution $w(x;\theta)$). So, in this situation we may use the theory of random variable transformation to get the p.d.f of the solution process if the joint p.d.f of the input variables are known. Returning to RVT theory [21], it requires that the number of random inputs to the transformation must equal the number of the outputs, so, here, we must make a simple mathematical modification to the problem to be compatible with the theorem. We introduce $(n-1)$ fictitious random outputs (Fig. 1) that take the form: