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Finite difference approximate solutions for a mixed sub-superlinear equation

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Abstract

In Ben Mabrouk and Ben Mohamed (2006) [A. Ben Mabrouk, M.L. Ben Mohamed, On some critical and slightly super-critical sub-superlinear equations. Far East J. Appl. Math. 23(1) (2006) 73–90, Special Volume of PDEs; A. Ben Mabrouk, M.L. Ben Mohamed, Nodal solutions for some nonlinear elliptic equations, Appl. Math. Comp., in press, doi:10.1016/j.amc.2006.08.003], it has proved some theoretical results dealing with some boundary value problem $\Delta u + f(u) = 0$ in B and $u = 0$ on ∂B, where B is a domain in $\mathbb{R}^d$ with smooth boundary. In the present paper, we perform a difference scheme method to approximate the solution of a nonlinear evolutionary problem associated to the elliptic problem studied in Ben Mabrouk and Ben Mohamed (2006). We give at the end some numerical implementations based on a mixed sublinear superlinear term of the form $f(u) = |u|^{p-1}u + \lambda|u|^{q-1}u$.

Keywords: Brezis–Nirenberg problem; Difference scheme; Parabolic problems; Error estimates

1. Introduction

Let Ω be a bounded domain in $\mathbb{R}^d$ and $T > 0$. Denote $\mathcal{Q} =]0, T] \times \Omega$. The paper is devoted to the study of some nonlinear evolutionary equation in a mixed sub-linear super-linear form derived from the following initial-boundary value problem:

$$\begin{cases} \partial_t u = A\Delta u + f(u), & (t,x) \in \mathcal{Q}, \\ u(t,x) = 0, & (t,x) \in]0,T] \times \partial\Omega, \\ u(0,x) = u_0(x), & x \in \Omega, \end{cases} \tag{1}$$

where u is a real valued vector-function, A a real diagonalisable matrix with positive eigenvalues, f a nonlinear function. ∂_t designs the partial derivative in time and Δ is the Laplace operator in $\mathbb{R}^d$. u_0 is a real valued

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vector-function which is usually supposed to be sufficiently regular. From the hypothesis on the matrix A, the above problem has no essential difference with the following mono-dimensional system:

$$\begin{cases} \partial_t u = a\Delta u + f(u), & (t,x) \in \mathcal{Q}, \\ u(t,x) = 0, & (t,x) \in]0,T] \times \partial\Omega, \\ u(0,x) = u_0(x), & x \in \Omega. \end{cases} \tag{2}$$

Here u is a real valued function, a is a positive real number.

In this paper, we will focus on some modified versions of problem (2). We introduce a mathematical model for a difference scheme discretization to the following Dirichlet conditions system in dimension 1:

$$\begin{cases} \partial_t u = a\partial_{xx} u + f(u), & (t,x) \in \mathcal{Q}, \\ u(t,0) = \varphi(t), & t \in]0,T], \\ u(t,1) = 0, & t \in]0,T], \\ u(0,x) = u_0(x), & x \in \Omega, \end{cases} \tag{3}$$

where $\Omega =]0,1[$. In higher dimensions, we will focus on the radial problem

$$\begin{cases} \partial_t u = a\partial_{rr} u + a\frac{d-1}{r}\partial_r u + f(u), & (t,r) \in]0,T]\times]0,1[, \\ u(t,0) = \varphi(t), & t \in]0,T], \\ u(t,1) = 0, & t \in]0,T], \\ u(0,r) = u_0(r), & r \in]0,1[. \end{cases} \tag{4}$$

Remark 1.1. Classical theory of PDEs shows that strong radial solutions of problem (2) satisfy $\partial_r u(t,0) = 0$.

Such problems has been the object of several studies in mathematics and physics such as nonlinear waves, chemical reaction models, reactor dynamics and heat conduction, Shrödinger equation when dealing with complex solutions and with $i\partial_t u$ instead of $\partial_t u, \ldots$. For more details and for a derivative and more general models, the reader can be referred to [8–16],... In the present work, we consider a real valued continuous function f satisfying the assumption

$$|f(u) - |u|^{p-1}u| \leqslant g(u), \tag{5}$$

where g is nonlipchitzian and locally q-Hölder continuous function, i.e.

$$|g(x) - g(y)| \leqslant C_{\text{loc}}|x - y|^q, \quad C_{\text{loc}}, \ q > 0. \tag{6}$$

We now explain the idea leading to this work and the novelty in our study. To do this we need to recall some recent works. An interesting case of the problem cited above is $f(u) = |u|^{p-1}u + \lambda|u|^{q-1}u$ with $0 < q < 1 < p$. Remark here that the function f is nonlipshitzian at zero but it is composed of concave and convex terms. Theoretically speaking, the nonlipshitzian behavior of f at the origin may yield some bad error estimates. The stationary case

$$\Delta u + f(u) = 0$$

with Dirichlet condition has been the object of some recent works by the first two authors. It derives from the famous equation of Brezis–Nirenberg

$$\begin{cases} \Delta u + u^p + \lambda u = 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \\ u > 0 & \text{in } \Omega, \end{cases} \tag{7}$$

where Ω is a bounded domain with smooth boundary in $\mathbb{R}^d$, $d \geqslant 3$ and p a real positive parameter number depending on a critical value $p_c = \frac{d+2}{d-2}$. Most of the recent studies were of a theoretical type and have been done for mainly classical bounded and positive solutions. Let λ_1 be the first eigenvalue of $-\Delta$ in $\Omega = B$ the unit ball