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Anticontrol of chaos of the fractional order modified van der Pol systems

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Abstract

Anticontrol of chaos of fractional order modified van der Pol systems is studied. Addition of a constant term and addition of $k|x|\sin x$ term where x is a state of the system are used to anticontrol the system effectively. By applying numerical results, phase portrait, Poincaré maps and bifurcation diagrams a variety of the phenomena of the chaotic motion can be presented. Finally, it can be find that chaos under these procedures exists in the fractional order systems of a modified van der Pol system.

Keywords: Chaos; Fractional order system; van der Pol equation; Modified van der Pol system; Anticontrol

1. Introduction

Anticontrol [1–7] and synchronization [8–16] of chaos have received great attention for many research activities in recent years. Anticontrol is an interesting, new and challenging phenomenon [17–19]. As a reverse process of suppressing or eliminating chaotic behaviors in order to reduce the complexity of an individual system or a coupled system, anticontrol of chaos aims at creating or enhancing the system complexity for some special applications. More precisely, anticontrolling chaos is to generate some chaotic behaviors from a given system, which is non-chaotic or even is stable originally. By fully exploiting the intrinsic non-linearity, this "control" technique provides another dimension for feedback systems design. Its potential applications can be easily found in many fields, including typically physics, biology, engineering, and medical as well as social sciences.

In addition, the topic of fractional calculus is enjoying growing interest not only among mathematicians, but also among physicists and engineers. In recent years, many scholars have devoted themselves to study the applications of the fractional order system to physics and engineering such as viscoelastic systems [20], dielectric polarization, and electromagnetic waves. More recently, there is a new trend to investigate the control [21] and dynamics [22–30] of the fractional order dynamical systems [31–34]. In [20] it has been shown that

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non-linear chaotic systems can still behave chaotically when their models become fractional. In [32,33], it was found that chaos exists in a fractional order Chen system with order less than 3.

In this paper, anticontrol of chaos of modified van der Pol systems [35–38] in fractional order form are studied. This paper is organized as follows. In Section 2, a fractional derivative and its approximation are introduced. In Section 3, a modified van der Pol system and the corresponding fractional order system are presented. In Section 4, numerical simulations are given. In Section 5, conclusions are drawn.

2. A fractional derivative and its approximation

There are several definitions of fractional derivatives. The commonly used definition for a general fractional derivative is the Riemann–Liouville definition [39], which is given by

$$\frac{\mathrm{d}^q f(t)}{\mathrm{d}t^q} = \frac{1}{\Gamma(n-q)} \frac{\mathrm{d}^n}{\mathrm{d}t^n} \int_0^t \frac{f(\tau)}{(t-\tau)^{q-n+1}} \, \mathrm{d}\tau, \tag{1}$$

where $\Gamma(\cdot)$ is the gamma function and n is an integer such that $n-1<q<n$. This definition is different from the usual intuitive definition of derivative. Fortunately, the basic engineering tool for analyzing linear systems, the Laplace transform, is still applicable and works as one would expect:

$$L\left\{\frac{\mathrm{d}^q f(t)}{\mathrm{d}t^q}\right\} = s^q L\{f(t)\} - \sum_{k=0}^{n-1} s^k \left[\frac{\mathrm{d}^{q-1-k} f(t)}{\mathrm{d}t^{q-1-k}}\right]_{t=0}, \quad \text{for all } q, \tag{2}$$

where n is an integer such that $n-1<q<n$. Upon considering the initial conditions to be zero, this formula reduces to the more expected form

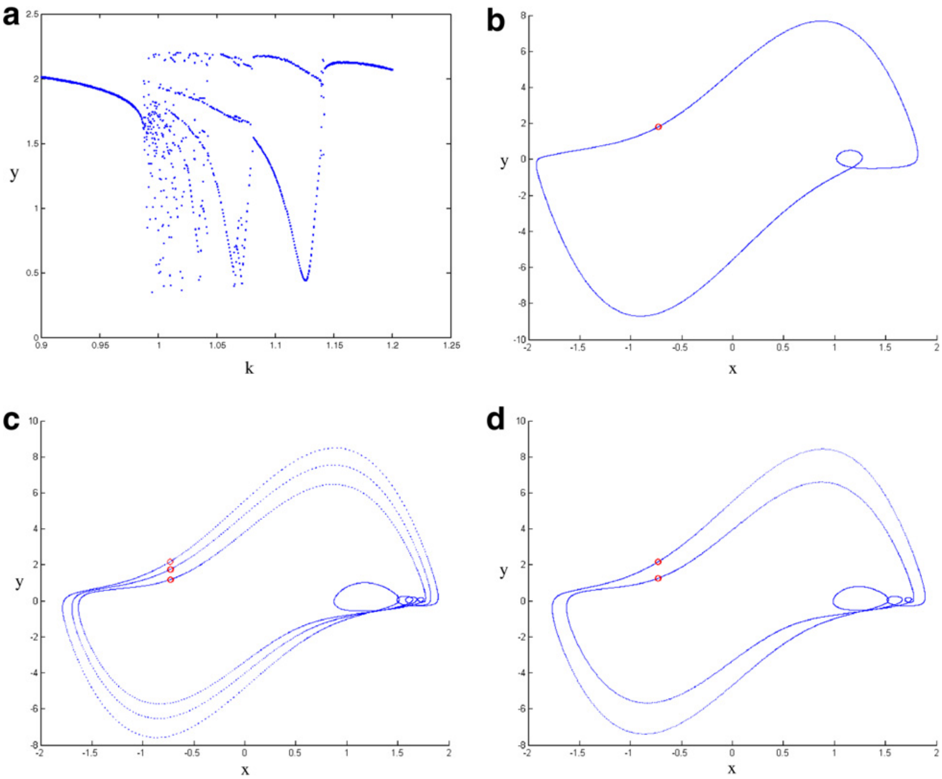

Fig. 1. (a) The bifurcation diagram for $\alpha=\beta=0.9$. (b) The phase portrait for $\alpha=\beta=0.9$, $k=0$. (c) The phase portrait for $\alpha=\beta=0.9$, $k=1.05$. (d) The phase portrait for $\alpha=\beta=0.9$, $k=1.1$.