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A integral filter algorithm for unconstrained global optimization

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Abstract

In this paper, making use of an integral inequality, a necessary and sufficient condition is given for a point to be a global minimizer. Based on the integral inequality, a novel integral-form algorithm is proposed for unconstrained global optimization. It is different from the other deterministic global search algorithm. Under mild conditions it is proved that, in theory, a global minimizer of the objective function can be certainly found by the presented algorithm. In order to indicate the efficiency and reliability of the method, four numerical examples are reported.

Keywords: Global optimization; Integral; Branch and bound; Local search algorithm

1. Introduction

Unconstrained global optimization is an important field in optimization. There are many papers devoted to global optimization in the open literatures, and many methods were introduced. In the deterministic methods, we often speak of the auxiliary function method and the branch and bound methods. These auxiliary function methods include the tunneling algorithm [1,2,11], filled function method [4,6,7,10] and so on. A common feature of these methods is that the subproblem of transcending local optimality, namely, given a local minimum x^*, we need to find a better feasible solution (i.e., escape from this local minimum x^*), or show that x^* is a global minimum. In the tunneling algorithm and the filled function method, a subproblem is constructed by replacing the objective function with an appropriately auxiliary function called the tunnelling function or the filled function, and then a local search procedure applied to the auxiliary function starting from x^* will lead to a lower minimum of the objective function (if there is any). However, these auxiliary functions depend critically on certain parameters whose correct values could only be determined by trials and errors, so there exist some difficulties in solving global optimization problems, and these methods do not have the convergence rule. The main idea of branch and bound method is that a global minimizer is found by gradually deleting some region where there is no global minimizers, see [5]. Although the branch and bound methods have

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the convergence rule, two tasks, i.e., branching and bounding, must be finished in these methods, so the optimization problem must have some special properties such that branching and bounding can proceed easily. Thus, for general nonlinearly global optimization problems, the branch and bound methods are not always efficient.

In this paper, making use of a integral inequality, a necessary and sufficient condition is given for a point to be a global minimizer. Based on the integral inequality, we propose a new integral-form algorithm which we called the integral filter algorithm for general unconstrained global optimization. In the process of algorithm, firstly, a local minimizer of the objective function is found by employing any local descent algorithm that starts from any initial point, and then, we construct a sequence of point to approach a point at which function value is less than the current local minimum, and such a point is obtained. Finally, we apply a local descent algorithm that starts from the point obtained to get a better minimizer of the objective function. Up to this point, we complete a circle of the algorithm. The foregoing circle goes on successively until the necessary and sufficient optimality condition is satisfied. In the procedure of constructing the sequence of points, we make use of the integral operation and successive bisection of feasible region. We also prove that the algorithm can find a global minimizer under mild conditions.

The paper is organized as follows: In Section 2, we state the problem under consideration, some assumptions and give the preliminary results. In Section 3, we describe an integral filter algorithm model and analyze its convergence properties. In Section 4, we report the numerical results on four examples. Finally, in Section 5, we give some concluding remarks.

2. Preliminary

We consider the following unconstrained programming problem:

$$\begin{aligned} &\min \quad f(x) \\ &\text{s.t.} \quad x \in R^n. \end{aligned}$$

Throughout this paper we make the following assumptions:

Assumption 1. $f(x)$ is Lipschitz continuous on R^n, i.e., there exists a constant $L > 0$ such that $|f(x) - f(y)| \leqslant L\|x - y\|$ holds for all $x, y \in R^n$.

Assumption 2. $f(x)$ is coercive, i.e., $f(x) \to +\infty$, as $\|x\| \to +\infty$.

Notice that Assumption 2 implies the existence of a robust compact set $\Omega \subset R^n$ whose interior contains all minimizers of $f(x)$. We assume that the value of $f(x)$ for x on the boundary of Ω is greater than the value of $f(x)$ for any x inside Ω. Then the original problem is equivalent to the following problem:

$$\begin{aligned} (\mathrm{P}) \quad &\min \quad f(x) \\ &\text{s.t.} \quad x \in \Omega. \end{aligned}$$

Assumption 3. $f(x)$ has only a finite number of minimums in Ω.

First, we recall the following integral inequality:

$$\int_D F(x)\mathrm{d}v \leqslant \int_D |F(x)|\mathrm{d}v,$$

where $D \subseteq R^n$ denotes the integral region, $\mathrm{d}v$ denotes the differential variable, and $F(x)$ is an integrable function. Obviously when $F(x) \geqslant 0$ for each $x \in D$, the equality sign is right. In addition, if $F(x)$ is continuous on D, then the equality sign holds if and only if $F(x) \geqslant 0$ for all $x \in D$. So we have the following theorem:

Theorem 2.1. *Suppose $F(x)$ is continuous in the region D. Let $D \in R^n$ be a robust compact set and $y \in D$ is a prefixed point. Then, there exists a point $x \in D$ such that $F(x) < F(y)$ if and only if*