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Analysis of three-dimensional grids: The estimation of two missing data

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Abstract

Two data that are missing from the eight- or nine-point prismatic array can often be estimated if the measurements can modeled by linear, quadratic, exponential, or trigonometric forms. The estimates are easily obtained and they are potentially useful when laboratory work is routine or expensive.

Keywords: Data replacement; Response surface; Experimental design; Operational equations; Prismatic array

1. Introduction

The expense of failed experiments or compromised data encourages the search for methods that replace lost measurements with estimates that are sufficiently accurate for routine purposes. This point has been illustrated by means of Eqs. (9)–(11) in Ref. [1]. Surrogate numbers for missing data are potentially useful when the law of data generation can be modeled by a linear expression or by simple quadratic, exponential, or trigonometric expressions. Although these requirements are not always satisfied in practice, interpolated surrogate numbers for missing data may justify the curtailment of laboratory work when the experiments are well understood or repetitious. Surrogate numbers for missing data may find economic justification if they can establish a history of acceptable accuracy [1].

2. Formulas for the eight- and nine-point prismatic arrays

A method for estimating two data that are missing from the eight-point prismatic array depends on Eq. (10) in Ref. [1]. The cited Eq. (10) is paired with Eq. (1) below and the two equations are solved as a set of simultaneous equations. Both equations are exact on the same functions as described in Ref. [1]. Letters in

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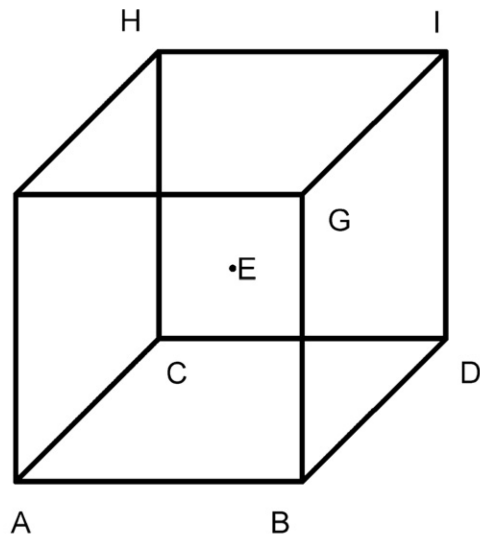

Fig. 1. The nine-point prism. Center point E is not used in eight-point designs.

the equations refer to numerical measurements obtained at the corresponding vertices of the prism shown in Fig. 1. A center point measurement is not required by Eq. (1) or the cited Eq. (10).

$$T_1 + T_2 + T_3 + T_4 + T_5 + T_6 + T_7 + T_8 + T_9 + T_{10} + 2T_{11} = 0, \tag{1}$$

$$T_1 = (I^2 + A^2)[3(CH + BG + HF + BD - 2BH) - FG - CD - 2(CG + FD)], \tag{2}$$

$$T_2 = 3(H^2 + B^2)(2AI - FA + FG - IG - ID + CD - CA), \tag{3}$$

$$T_3 = C^2[3(HF - AF - IH) + 2IA + ID - BD + BA], \tag{4}$$

$$T_4 = D^2[3(BG - BA - IG) + CA - CH + IH + 2IA], \tag{5}$$

$$T_5 = F^2[3(CH - CA - IH) + BA - BG + IG + 2IA], \tag{6}$$

$$T_6 = G^2[3(BD - ID - AB) + 2AI + FA + IH - HF], \tag{7}$$

$$T_7 = 2AB[3(HG + HD - GD - IF - CI) + 5CF], \tag{8}$$

$$T_8 = 2CD(AG + IF - BI - BF - HG - HA), \tag{9}$$

$$T_9 = 2FG(AD - HD - HA + CI - BC - BI), \tag{10}$$

$$T_{10} = 2IH[5GD + 3(BF + BC - AG - CF - AD)], \tag{11}$$

$$T_{11} = CH(AG - 3BF) + IA(GD + CF) + BG(CI - 3DH) + DF(BI + AH). \tag{12}$$

Let the eight-point cube in Fig. 1 be substituted with the integers $1,\ldots,4$, and $6,\ldots,9$ at vertices $A,\ldots,D$ and $F,\ldots,I$, respectively. The cube can be rotated so that the integers 8 and 4 appear at vertices D and H. If these two numbers are deleted from the rotated cube, the remaining data are $[7,6,9,2,1,3]$ at vertices $[A,B,C,F,G,I]$, respectively.

Monotonic functions can be applied to $[7,6,9,2,1,3]$ and the results substituted into Eqs. (1) and (10) in Ref. [1] as $[A,B,C,F,G,I]$, respectively. Typical monotonic functions are listed in the first column of Table 1. Substituted Eqs. (1) and (10) in Ref. [1] are solved simultaneously to estimate the missing data at vertices D and H in Fig. 1.

The described procedure commonly yields more than one set of two estimates for the two missing data. Selected estimates rendered by the set of simultaneous equations appear in the second column of Table 1. The third column of the table lists the true values as obtained by applying the trial functions to the integers 8 and 4, respectively. The entries in Table 1 suggest that the simultaneous equations yield recognizable estimates of the two missing data in many simple cases [1].