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An extension of Newton–Raphson power flow problem

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Abstract

This paper explores an idea to extend Newton–Raphson power flow problem to handle power system transmission line flow limits, by means of generation redispatch and phase shifters. We extend and reformulate the power flow so that it includes a variety of flow limits (thermal, small-signal stability, voltage difference), generation redispatch, and phase shifters. The novelty of the approach is three step procedure (in case any limit violations exist in the system): run ordinary power flow (and identify flow limits violated), solve a set of linear equations using extended power flow Jacobian by adding a new column and a new raw that characterize particular limit, and resolve ordinary power flow with initial solution obtained after the correction made by solution of linear equations. The use of ordinary power flow Jacobian and minimal extensions to it in the case of limits identified makes this approach an attractive alternative for practical use. A simple numerical example and the examples using an approximate model of real-life European Interconnected Power System are included in the paper to illustrate the concept.

Keywords: Newton–Raphson power flow; Power system; Generator redispatch; Phase shifters; Flow limits

1. Introduction

The steady-state conditions of a power system are typically modeled, from Kirchoff laws and power conservation, by a system of non-linear algebraic equations [1–3]:

$$\mathbf{f}(\mathbf{x}) = \mathbf{0}, \tag{1}$$

where $\mathbf{x} \in \Re^n$ is the vector of state variables (bus voltage magnitudes and phase angles), and $\mathbf{f} : \Re^n \rightarrow \Re$ is sufficiently smooth function. Ordinary power flow problem [1,2] for a system with N buses (out of which N_g are generation buses) consists of the solution of $n = N + (N - N_g + 1)$ simultaneous non-linear equations with n unknowns. The equations that are solved simultaneously include: an active power balance equation ΔP_i for

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Nomenclature

θ	bus voltage phase angle (rad)
V	bus voltage magnitude (V)
P	active power (MW)
Q	reactive power (MVAr)
S	apparent power (MVA)
I	current magnitude (A)
Y	admittance (Ω)
G	real part of Y (Ω)
B	imaginary part of Y (Ω)
$\mathbf{J}$	Jacobian matrix
$\mathbf{J}_\mathrm{f}$	flow Jacobian matrix
$\mathbf{S}_m$	sensitivity matrix
P^r	redispatch amount (MW)
α	phase shift angle (rad)

every bus except one bus that is designated as the "slack" bus (the bus so designated is always one of the generation buses), and a reactive power balance equation ΔQ_i for every load bus (buses where both P and Q injections are specified). There are many extensions of the ordinary power flow problem aiming either to ease computational burden or to include some realistic limits such as power generation (both active and reactive power) or bus voltage magnitude limits [1,2]. Consideration of realistic physical power system limits is of paramount importance in order to ensure physical feasibility of the solution.

The factors that influence the limiting values of transmission line flows are [4]: thermal limit (I_f^2 limit, I stands for current while f stands for flow), small-signal stability limit (P_f limit), and voltage difference limit (S_f limit, S stands for apparent power). Both generation redispatch and phase shifters have been recognized as useful means to handle line flow limits. For some early considerations interested readers are referred to [5–9] and for recent ones to [10–13]. All these considerations could be roughly classified as optimization [8,13] and non-optimization (direct) based [5–7,9–12]. Optimization based methods (particularly optimal power flow) are arguably more accurate but computationally expensive and time consuming [11]. Direct methods emerged as the need for an efficient and fast method that trade optimality for effectiveness, so that power system operators can make quick yet efficient decisions under stressed conditions of the power system. The method developed in this paper belong to direct methods and differs from previous similar considerations in the way how ordinary power flow is extended and reformulated in order to include a variety of flow limits, generation redispatch and phase shifters. The novelty of the approach is three step procedure (in case any limit violations exist in the system): run ordinary power flow (and identify flow limits violated), solve a set of linear equations using extended power flow Jacobian by adding a new column and a new raw that characterize particular limit, and resolve ordinary power flow with initial solution obtained after the correction made by solution of linear equations. The use of ordinary power flow Jacobian and minimal extensions to it in the case of limits identified makes this approach an attractive alternative for practical use. To facilitate the presentation throughout of the paper we focus on active power flow limits and in the appendix provide generic derivations for all three types of line flow limits. We extend the fast vectorized version of ordinary power flow, implemented in MATLAB® environment [2,3].

The paper is organized as follows. In Section 2 ordinary power flow problem is reviewed and possible extensions discussed, Section 3 describes how flow limits can be relieved by generation redispatch and provides some analytical results on the reformulation of ordinary power flow problem. Possibilities of using phase-shifting transformers for the same purpose are presented in Section 4. Section 5 provides the results obtained with help of real-life European Interconnected power system, while Section 6 offers some conclusions.