Available online at www.sciencedirect.com
ScienceDirect
Applied Mathematics and Computation 186 (2007) 1263–1275
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

Analysis of OWA operators in decision making for modelling the majority concept

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Abstract

The majority concept plays a main role in decision making processes where one of the main problems is to define a decision strategy which takes into account the individual opinions of the decision makers to produce an overall opinion which synthesizes the opinions of the majority of the decision makers. The reduction of the individual values into a representative value of majority is usually performed trough an aggregation process. The most common operator used in these processes is the OWA operator, in which the majority concept can be modelled using fuzzy logic and linguistic quantifiers. In this work the fusion processes and the semantic used for modelling the majority concept in the OWA operators are analyzed and compared in order to present different approach to obtain a feasible majority aggregation value for the decision making problem.

Keywords: OWA; IOWA; MA-OWA operators; Linguistic quantifiers; Group decision making; Majority opinion

1. Introduction

Decision making is a usual task in human activities where a set of experts work in a decision process to obtain a final value which is representative of the group. The first step of this decision process is constituted by the individual evaluations of the experts; each decision maker rates each alternative on the basis of an adopted evaluation scheme [1–3]. We assume that at the end of this step each alternative has associated a performance judgment on the linguistic scale (or numeric scale). The second step consists in determining for each alternative a consensual value which synthesizes the individual evaluation. This value must be representative of a collective estimation and is obtained by the aggregation of the opinions of the experts [3–9]. Finally, the process concludes with the selection of the best alternative/s as the most representative value of solution of the problem.

One of the main problems in decision making is how to define a fusion method which considers the majority opinions from the individual opinions. To obtain a value of synthesis of the alternatives which is representative

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0096-3003/$ - see front matter
doi:10.1016/j.amc.2006.07.161

of the opinions of the experts exist diverse approaches in which are realized an aggregation guided by the concept of majority, where majority is defined as a collective evaluation in which the opinions of the most of the experts involved in the decision problem are considered [8]. In these approaches the result is not necessarily of unanimity, but it must be obtained a solution with agreement among a fuzzy majority of the decision makers [7,10,11].

In the fuzzy approaches to decision making the concept of majority is usually modelled by means of linguistic quantifiers such as *at least 80%* and *most*. A linguistic quantifier is formally defined as a fuzzy subset of a numeric domain [12,13]. The definition of the linguistic quantifier *most* appear in Eq. (1). The semantics of such a fuzzy subset is described by a membership function which describes the compatibility of a given absolute or percentage quantity to the concept expressed by the linguistic quantifiers. By this interpretation a linguistic quantifier is seen as a fuzzy concept referred to the quantity of elements of a considered reference set

$$Q_{\text{most}}(x) = \begin{cases} 1 & x \geqslant 0.9, \\ 2x - 0.8 & 0.4 < x < 0.9, \\ 0 & x \leqslant 0.4. \end{cases} \tag{1}$$

In group decision making, linguistic quantifiers are used to indicate a fusion strategy to guide the process of aggregating the experts' opinions [11]. The results of this aggregation process must represent the semantic of the linguistic quantifier. An example of linguistic expression which employs a quantifier guided aggregation is the following: Q experts are satisfied by solution a, where Q denotes a linguistic quantifier (for example *most*) which expresses a majority concept. To produce a solution which satisfies this proposition the experts' opinions must be aggregate using an operator which captures the semantics of the concept expressed by the quantifier Q.

In this paper the problem of constructing a majority opinion using quantifiers and OWA operators is considered and the semantics of the performed aggregation is analyzed. In particular we observe that the usual definition of OWA operators based on linguistic quantifiers does not capture a semantics of a consensus of the majority which is typical of decision making, for this reason we analyze and compare the most common OWA operators used in decision making problems in order to present different approach to obtain a feasible majority aggregation value for the decision making problem.

The paper is structured as follows: in Section 2 the aggregation guide by quantifiers in OWA operators are introduced and the problems for modelling the majority semantic are explained. In Section 3 some variations of OWA operators are analyzed; in Section 4, two majority fusion strategies for MA-OWA operators for modelling the majority concept are defined. Finally the conclusions are exposed.

2. OWA operators

An OWA operator [9] is defined as a mapping function $F: R^n \to R$ that has associated a weighting vector W with length n

$$W = [w_1, w_2, \ldots, w_n]^{\mathrm{T}}.$$

Such as $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$

$$\mathrm{OWA}(a_1, a_2, \ldots, a_n) = \sum_{i=1}^{n} w_i \cdot b_i$$

with b_i being the ith largest element of the a_j.

Furthermore, if W is a vector whose components are w_i, and B is a vector whose components are the ordered arguments values a_j, then

$$\mathrm{OWA}(a_1, a_2, \ldots, a_n) = W^{\mathrm{T}} \cdot B.$$

A fundamental aspect of these operators is the reorder step of the arguments. This produces that the element to aggregate a_i is not associated with a weight w_i, but a weight w_i will be associated with an ordered position in the aggregation.