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Enhancing Poincare plot information via sampling rates

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Abstract

Poincare plots of a continuous solution of the logistic equation at different sampling rates are studied. The study uses the continuous solution obtained for the parameter $A = 4$ in the logistic equation. This solution of the logistic equation used in this study provides a useful handle to study sampling effects and at the same time look at a system which exhibits complex correlations. The results of this study show that the Poincare plots of a continuous function at different sampling rates provide valuable information about the complex correlations present in the system. This additional information provided by the Poincare plots of data from a continuous system sampled at different rates will be a useful and simple way of studying correlations present in a real system with applications in biomedicine.

Keywords: Poincare plot; Sampling rate; Logistic equation; Correlations

1. Introduction

Poincare plot [1] is a technique taken from nonlinear dynamics, which takes a sequence of sample values and plots each sample against the following sample. Statistically it displays the correlation between consecutive intervals in a graphical manner. But the significance of this plot is that it is the two dimensional reconstructed phase space, the projection of the reconstructed attractor that describes dynamics of the time series [2]. It has been used to display visually biomedical signals [3,4] and conclusions are drawn on the variability and shape of these plots.

In the construction of Poincare plots of a real system, digitized sample values obtained from a continuous system are used. The issue that is being addressed in this paper is the dependence of the Poincare plot on the sampling interval of the time series, with a view to understanding the correlations present in the system. Can we understand more about a dynamical system by constructing Poincare plots of the continuous signal sampled at different rates?

In order to examine this, a continuous and the discrete solution of the logistic equation are studied. The logistic equation is given by [5],

$$x_{n+1} = Ax_{n-1}(1 - x_{n-1}), \tag{1}$$

where A is a constant. For increasing values of A, the equation progresses from a single-value convergence to chaos. Eq. (1) is recursive and the series is evaluated for integer values of n using an initial value x_0 which lies

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between 0 and 1. This function exhibits chaotic behaviour for $A > 3.57$. However for $A = 4$, which falls within the region of chaos, there is an exact solution. This provides us an opportunity to have a solution where one can vary the sampling rates and a solution which exhibits chaotic behaviour when the sampling interval is unity. In this study, both the exact solution along with the recursive solution from (1) for $A = 4$, is used.

In Section 2, the exact solution of logistic equation of $A = 4$ is described and its solution compared with the recursive solution. Section 3, shows the different Poincare plots that are obtained when the sampling rates are changed and Section 4 is the conclusion.

2. Exact solution of the logistic equation for $A = 4$ and comparison with the recursive solution

The solution of the exact solution of Eq. (1) for $A = 4$, valid for all real values of n is given by [6],

$$x_n = 0.5[1 - T_{2^n}(1 - 2x_0)], \tag{2}$$

where $T_n(y)$ is a Chebyshev function [6], valid for all real n.

Fig. 1 compares the exact solution (2) evaluated at intervals of $\Delta n = 5 * 10(-5)$ with the exact solution evaluated at $\Delta n = 1$. The time series for the two sampling times are very different. They are evaluated with $x_0 = 0.2$. The solution for $\Delta n = 5 * 10(-5)$ shows a well behaved oscillatory behaviour, where the frequency increases with n, while the solution for $\Delta n = 1$ appears more random. Fig. 2 compares the exact solution evaluated at $\Delta n = 1$ with the recursive solution of Eq. (1). Although the recursive solution agrees with the exact solution for small n, as n increases due to propagation error in the recursive solution, differences emerge. In Fig. 3, the Poincare plot of the exact solution evaluated at $\Delta n = 1$ is compared with the recursive solution of Eq. (1). The result shows there is no difference in the geometry of the Poincare plot. Although the time series for both cases as seen in Fig. 2 do not show exact agreement, the shape of the Poincare plots are the same, showing that the propagation errors in the recursive solution do not effect in the shape of the Poincare plot.

3. Poincare plots for different sampling rates

In this section the results of the Poincare plots are shown for different sampling rates. Fig. 4 shows the Poincare plots of the continuous solution for sampling intervals $\Delta n = 5.0 * 10(-5)$, $10(-4)$, $10(-3)$, $10(-2)$, $10(-1)$

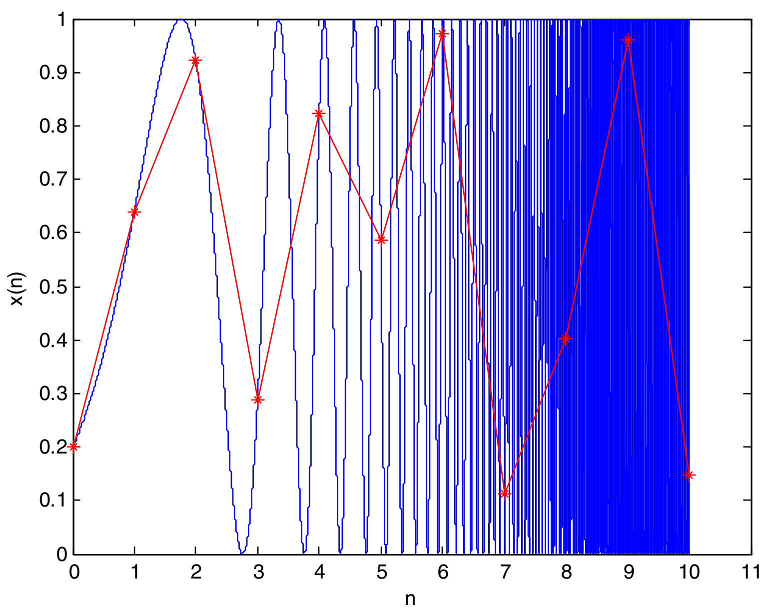

Fig. 1. Comparison of the exact solution at intervals of $\Delta n = 5 * 10(-5)$ (blue line), with the exact solution at $\Delta n = 1$. (For interpretation of the references in colour in this figure legend, the reader is referred to the web version of this article.)