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Wavelet moment method for solving Fredholm integral equations of the first kind

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Abstract

In this paper, we suggest an efficient method for solving Fredholm integral equations of the first kind, using wavelets as basis functions in the moment method and reducing the order of the linear equation, rather than making the matrix sparse. In the end, we give some numerical examples.

Keywords: Fredholm integral equations of the first kind; Wavelets; Moment method; Linear system

1. Introduction

We know that the integral equations of the first kind are among ill-posed problems, [1–6], moreover Fredholm integral equation of the first kind arises in many problems in engineering fields. So the study of such problems is very useful in application. Several methods have been proposed, such as expansion method, [2,7,8], regularization method, [2–4,9], Galerkin method [2,7], and other methods, [5,10–12]. But the method that we suggest depend on moment method which plays an important role in numerical methods and numerical computations, [13,14]. In general, the coefficient matrix of moment method is dense, and solution of corresponding system of linear equations is very time consuming or the run time is high. The major computational difficulty in implementing Galerkin method is that virtually for all practical cases the inner products need to evaluated numerically. In particular the task of evaluating the double integrals involved, can be quite difficult. To overcome the difficulties of enormous memory requirement and high cost of computation run time, many researchers have proposed the use of wavelet basis, [15–17].

As we know, it is important to select a suitable basis function in numerical methods for integral equations. Many kinds of basis functions have been proposed, such as triangular basis function, pulse basis function,

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polynomial basis function, spline and B-spline basis function. Recently, wavelet basis function has been proposed to solve Fredholm integral equations numerically, [18,19].

Because of the local supports and vanishing moment properties of wavelets, [15,16], most of the corresponding matrix elements are very small compared to the largest element, and hence can be neglected without significantly affecting the solution, [18,19]. Using moment methods and subsequently a threshold procedure, the matrix constructed by these methods can be rendered sparse, [19]. Then, a system of linear equations with sparse matrix is solved.

2. Wavelet moment method

In this section, we describe wavelet moment method for solving Fredholm integral equation of the first kind. The basis functions are wavelets defined on the interval $[0,1]$. We consider the integral equation of the form

$$\int_0^1 k(s,t)x(t)\mathrm{d}t = y(s), \quad s \in [0,\, 1]. \tag{1}$$

We know [15,14], any function $x(s) \in L[0,1]$ can be written as

$$x(s) = \sum_{i=0}^{n} x_{0,i}\phi_i(s) + \sum_{j=1}^{\infty}\sum_{k=1} 2^{j-1}nx_{j,k}\psi_{j,k}(s). \tag{2}$$

In the wavelet moment method, we choose

$$\{\phi_i(s),\, \psi_{j,k}(s)\},$$

as the basis and testing functions. For approximation, we must select a finite set of the basis functions, and approximate $x(s)$ by $x_N(s)$, as below:

$$x(s) \approx x_N(s) = \sum_{i=0}^{n} x_{0,i}\phi_i(s) + \sum_{j=1}^{N}\sum_{k=1}^{2^{j-1}n} x_{j,k}\psi_{j,k}(s), \tag{3}$$

where n and N, denote number of scales and order of approximation, respectively.

By inserting (3) in (1) and neglecting the error, we have

$$\sum_{i=0}^{n} x_{0,i}\int_0^1 k(s,t)\phi_i(t)\mathrm{d}t + \sum_{j=1}^{N}\sum_{k=1}^{2^{j-1}n} x_{j,k}\int_0^1 k(s,t)\psi_{j,k}(t)\mathrm{d}t = y(s). \tag{4}$$

Now, by considering the set $\{\phi_i(s), \psi_{j,k}(s)\}$ as basis functions, and use the linearity of the inner product we have the following equations, known as wavelet moment method equations:

$$\sum_{i=0}^{n} x_{0,i}\int_0^1\int_0^1 k(s,t)\phi_i(t)\phi_m(s)\mathrm{d}t\mathrm{d}s + \sum_{j=1}^{N}\sum_{k=1}^{2^{j-1}n} x_{j,k}\int_0^1\int_0^1 k(s,t)\psi_{j,k}(t)\phi_m(s)\mathrm{d}t\mathrm{d}s$$
$$= \int_0^1 y(s)\phi_m(s)\mathrm{d}s, \quad m = 0,1,2,\ldots,n,$$

and

$$\sum_{i=0}^{n} x_{0,i}\int_0^1\int_0^1 k(s,t)\phi_i(t)\psi_{l,h}(s)\mathrm{d}t\mathrm{d}s + \sum_{j=1}^{N}\sum_{k=1}^{2^{j-1}n} x_{j,k}\int_0^1\int_0^1 k(s,t)\psi_{j,k}(t)\psi_{l,h}(s)\mathrm{d}t\mathrm{d}s$$
$$= \int_0^1 y(s)\psi_{l,h}(s)\mathrm{d}s, \quad l = 1,2,\ldots,N, \;\; h = 1,2,\ldots,2^{l-1}n.$$

Now, we write the above equations in the matrix form

$$\mathbf{AX} = \mathbf{Y}$$