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The approximation algorithm for solving a sort of non-smooth programming

Hou Zai-En [a,*], Duan Fu-Jian [b]

[a] *Shaanxi University of Science and Technology, Xianyang 712081, PR China*
[b] *Guilin University of Electronic Technology, Guilin 541004, PR China*

Abstract

An approximation algorithm for solving a sort of non-smooth programming is proposed. In the programming, the objective function is the Hölder function and the feasible region is contained in a rectangle (viz. hyper-rectangle). To establish the algorithm, the properties of the Hölder function and an approximation of the function by using Bernstein α-polynomial are studied. According to the properties of the approximation polynomial and the algorithm for solving geometric programming, the strategy for branching and bounding and the branch-and-bound algorithm are constructed to solve the programming. The convergence of the algorithm can be guaranteed by the exhaustive of the bisection of the rectangle. The feasibility of the algorithm is validated by solving an example.

Keywords: Hölder function; Non-smooth programming; Approximation polynomial; Geometrical programming; Branch-and-bound algorithm

1. Introduction

The following model is called as non-smooth programming

$$(\text{NSP})\begin{cases} \min & f(x) \\ \text{s.t.} & g_i(x) \leqslant 0 \quad (i=1,2,\ldots,m), \\ & h_j(x)=0 \quad (j=1,2,\ldots,n), \end{cases}$$

where there exists one non-smooth function in $f(x)$, $g_i(x)(i=1,2,\ldots,m)$ and $h_j(x)(j=1,2,\ldots,n)$. The study for solving the programming has attracted much attention since the programming is widely existed in practical engineering and the penalty function of a constrained smooth programming is generally a non-smooth function. According to the properties of the programming and the study on smooth programming, the necessary and sufficient conditions and the basic methods are established to solve the programming (see [1–3]). And some modern methods for solving smooth programming are applied to solve the programming, such as bundle

* Corresponding author.
E-mail address: hkze-00795@163.com (Z.-E. Hou).

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method, trust region method, reformed Newton method and SQP method, which is a successive approximation method with quadratic programming (see [4–9]). Some of these methods has requests or restrictions on smoothness or convexity to the function in the programming. The typical example is the Lipschitz optimization in which the Lipschitzian continuity is assumed to the objective and constraint function (see [10,11]). Can these restrictions be weakened in model (NSP)? These motivate us to study the properties of the Hölder function, the approximation of the function by using Bernstein α-polynomial and the strategy for branching and bounding. Our goal is to develop a new method for solving the programming where the objective function and constrained functions are all Hölder continues (see [12]).

The content of this paper is as follows. In Section 2 we review the properties of the Hölder function and the approximation of the function by using Bernstein α-polynomial. In Section 3 we construct the algorithm and study the convergence of the algorithm for solving the programming. In Section 4 we give a numerical example to illuminate the feasibility of the algorithm. We end the paper with the conclusion of the paper in Section 5.

2. The properties of Hölder function

2.1. Hölder function

In this section we review the concept of Hölder function and study several properties of the function.

Definition 1. Let $f(x)$ be a real function on $P(\subset \mathbb{R}^n)$, $f(x)$ is called a Hölder function on P (or Hölder continuous) if there exist constant $L = L(f, P) > 0$ and $\gamma > 0$ such that

$$|f(x_2) - f(x_1)| \leqslant L\|x_2 - x_1\|^{\gamma} \quad \text{for all } x_1, x_2 \in P, \tag{1}$$

where constants $L = L(f, P) > 0$ and $\gamma > 0$ are called Hölder constants of $f(x)$.

In the definition the norm $\|\cdot\|$ is the general Euclidean norm. In practice, the following l_p- norm is often adopted

$$\|x\|_p = \left(\sum_{i=1}^{s} |x_i|^p\right)^{\frac{1}{p}} \quad (1 \leqslant p \leqslant \infty), \tag{2}$$

where $\|x\|_\infty = \max_{i=1,\dots,s}|x_i|$.

Obviously, Hölder function $f(x)$ is a Lipshchitz function on P when $\gamma = 1$, that is, Lipshchitz function is a special kind of Hölder function. Even so, the continuity of them is alike, and other properties of them are analogous as well.

Lemma 2. *Let $f(x), g(x), f_i(x)$ $(i = 1, \dots, m)$ be Hölder functions on the compact set $P \subset \mathbb{R}^s$, then for any $x \in P$, there exists a neighborhood $U(x)$ at x, such that*

(i) *Every linear combination of $f_i(x)$ $(i = 1, \dots, m)$ is a Hölder function on $U(x) \cap P$;*
(ii) $\max_{i=1,\dots,m} f_i(x)$ *and* $\min_{i=1,\dots,m} f_i(x)$ *are Hölder functions on $U(x) \cap P$;*
(iii) *$f(x)\cdot g(x)$ is a Hölder function on $U(x) \cap P$.*

Proof. The constructive method may be used in the proofs of (i)–(iii). The proof of $\max_{i=1,\dots,m} f_i(x)$ in (ii) is given here, the others are similar.

Suppose L_i and γ_i $(i = 1, \dots, m)$ are Hölder constants of function $f_i(x)$ $(i = 1, \dots, m)$, respectively, by the Definition 1, we have following inequalities:

$$|f_i(x_2) - f_i(x_1)| \leqslant L_i\|x_2 - x_1\|^{\gamma_i} \quad \text{for all } x_1, x_2 \in P\,(i = 1, \dots, m). \tag{3}$$

Let

$$F(x) = \max_{i=1,\dots,m} f_i(x) \tag{4}$$