Available online at www.sciencedirect.com
ScienceDirect
Applied Mathematics and Computation 186 (2007) 1694–1700
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

Analysis of three-dimensional grids: Interpolating the nine-point prism. The shifting operator as an analytical instrument

G.L. Silver

Los Alamos National Laboratory[1] P.O. Box 1663, MS E517, Los Alamos, NM 87545, United States

Abstract

A new equation for interpolating the nine-point prism by the circular or hyperbolic functions is illustrated. It is useful for estimating linear-term coefficients. New formulas for analytic and numerical differentiation and integration are obtained by the shifting operator. They are illustrated by examples.

Keywords: Response surface; Experimental design; Cube; Differentiation; Operational equations

1. Introduction

Previous papers have illustrated operational equations for interpolating the nine-point prismatic array by sines and cosines. One of the equations uses the center point datum only for estimating the numerical coefficients that appear outside the cited functions [1]. The second equation uses the center point datum only in the denominators of expressions that estimate the numerical coefficients appearing as arguments of the sine and cosine [2]. This paper illustrates a method for incorporating the center point datum in both the numerators and denominators of the same expressions.

Most textbooks do not illustrate the application of the shifting operator for analytic differentiation. This paper illustrates how the operator can be used to obtain a formula for that purpose. An operational formula for analytic integration is described. Textbooks of numerical analysis commonly include a formula for the numerical estimation of the first derivative at the center point of three equidistant, curvilinear data. The familiar formula is shown to be one member of a family of first derivative expressions that are based on the shifting operator. A new formula for numerical integration on three equidistant, curvilinear data is illustrated.

E-mail address: gsilver@lanl.gov
[1] Los Alamos National Laboratory is operated by the University of California for the US Department of Energy under Contract No. W-7405-ENG-36.

0096-3003/$ - see front matter
doi:10.1016/j.amc.2006.08.076

2. Sine-and-cosine equation for the nine-point prism

An equation for interpolating the nine-point prism, shown in Fig. 1, is based on the circular or hyperbolic sines and cosines. It has been illustrated by Method 1 in Ref. [1]. In that equation, the center point datum does not affect the numerical coefficients that appear as arguments of the sine and the cosine. Ref. [2] illustrates another interpolating equation for the same design. In the second equation, the arguments of the sine and cosine are denoted p, q, and r. The numerical values of these three parameters are estimated by means of ratios that are the arguments of the inverse hyperbolic cosine. The relationships are illustrated by Eqs. (1)–(3). Single letters denote numbers located at the corresponding vertices in Fig. 1. Four-letter designations denote interpolated numbers at the center points of the six prism faces in the same figure.

$$p = \text{arccosh}((\text{BDIG} + \text{ACHF})/2\text{E}), \tag{1}$$

$$q = \text{arccosh}((\text{ABGF} + \text{CDIH})/2\text{E}), \tag{2}$$

$$r = \text{arccosh}((\text{ABDC} + \text{FGIH})/2\text{E}). \tag{3}$$

The interpolating equation illustrated in Ref. [2] uses the center point datum only in the denominators of the ratios that determine the numerical values of p, q, and r. The interpolated responses at the center points of the prism faces are determined by the numbers at the eight corner points of the prism. An alternative equation applies E in both the numerators and the denominators of the ratios in Eqs. (1)–(3) [2].

Eq. (4) estimates the response at the center point of face ACHF in Fig. 1. It depends on all nine measurements A–I in the prism. Eq. (4) is an even-power expression so the interpolating equation derived from it is ordinarily limited to positive data. Rotating the prism and reapplying Eq. (4) yields numerical estimates at the center points of the remaining five faces of the cube. The numerical values of p, q, and r in Eqs. (1)–(3) are evaluated by means of Eq. (4). The interpolating equation for the prism is then completed as described in Refs. [1,2].

$$\begin{aligned}(\text{ACHF})^4 = {} & (\text{F}+\text{C})(\text{A}+\text{H})[2(\text{C}+\text{D}-\text{F}-\text{G})\text{E}^2 + \text{F}(\text{C}^2+\text{DG}-\text{CF}) - \text{CDG}][2(\text{A}+\text{B}-\text{I}-\text{H})\text{E}^2 \\ & + \text{A}(\text{AH}-\text{BI}-\text{H}^2) + \text{HBI}]/[(\text{H}+\text{I}+\text{A}+\text{B})(\text{H}+\text{I}-\text{A}-\text{B}) \\ & \times (\text{F}+\text{G}+\text{C}+\text{D})(\text{F}+\text{G}-\text{C}-\text{D})].\end{aligned} \tag{4}$$

For example, let the data be the third powers of the first nine integers $(1,2,3,\ldots,7,8,9)$ as $(\text{A},\text{B},\text{C},\ldots,\text{G},\text{H},\text{I})$ in Fig. 1, respectively. The numerical equation interpolating them is Eq. (5). It can be compared to Eq. (1) in Ref. [1] and Eq. (9) in Ref. [2]. All of the cited equations are invariant under rotation of the prism but not under its translation. Table 1 lists the sums of the squares of the deviations of interpolating equations like Eqs. (5) and (9) in Ref. [2] from typical monotonic test surfaces. Similar data are listed for the trilinear equation for purposes of comparison.

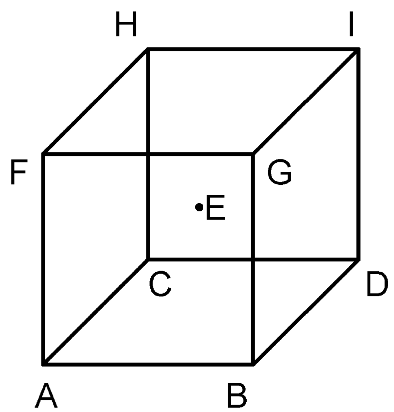

Fig. 1. The nine-point prism.